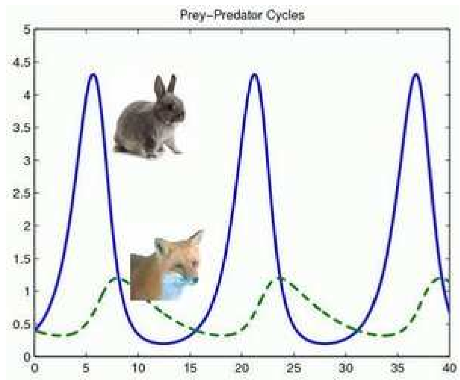
Eastern Oregon University
Project
MATH 323 - Mathematical Modeling



1 Project Overview

The basic goal of the project is to determine the examine predator-prey populations. As a model, you are to use the following set of differential equations:

$$\frac{dG}{dt} = -a_{hg}HG + b_g G\left(1 - \frac{G}{G_{eq}}\right) \tag{1}$$

$$\frac{dH}{dt} = -a_{ch}CH + \left(\frac{a_{gh}G}{1 + \frac{a_{gh}}{b_h}G}\right)H - b_{h2}H \tag{2}$$

$$\frac{dC}{dt} = a_{hc}HC - b_c C \tag{3}$$

The dependent variables are number of servings of food, $G(t)$ (Grass), the number of herbivores, $H(t)$, and the number of carnivores, $C(t)$. As initial conditions, we will imagine that there are 1000 servings of Grass, 200 Herbivores, and 25 Carnivores.

1. You will use the following information to determine the constants in the model:

   (a) In the absence of predation, the grass population with equilibrate to 2000 servings.

   (b) Whenever there is a Grass-Herbivore encounter 1 grass serving is eaten. These encounter occur once per day when there is one Herbivore and one Grass Serving.

(c) In the absence of predation, a small amount of grass will double its size in 20 days.

(d) In the absence of predation and food sources, 90% the herbivores will die in 60 days.

(e) In the absence of predation with large food supplies, the herbivores will double its population in 180 days.

(f) $a_{gh} = \frac{1}{3600}$

(g) For every 10 Carnivore-Herbivore encounters, 3 Herbivores are eaten. These encounter occur once per 7 days when there is one Herbivore and one Grass Serving.

(h) $a_{hc} = \frac{1}{9600}$

(i) In the absence of food sources, 90% the carnivores will die in 30 days.

2. Determine all of the fixed point populations.

3. Using 4^{th} order Runge-Kutta, determine what will happen to the grass in the absence of herbivores and carnivores. Make a plot of your results.

4. Using 4^{th} order Runge-Kutta, determine what will happen to the grass and herbivores in the absense of carnivores. On one graph, make a plot of herbivore and grass populations as a function of time. Also, make a phase plot of herbivore population vs. grass population.

5. Using 4^{th} order Runge-Kutta, determine what will happen to each of the populations. On one graph, make a plot of each of the populations as a function of time. Also, make all three possible phase plots.

You need to demonstrate the use of

- Runge-Kutta
- a spreadsheet using the Euler Method.
- exact solutions to the ODE where possible.

2 The Report

Your report should follow this format closely:

1. Title Page
   (a) Title
   (b) Abstract
2. Introduction
   (a) History of Population Modeling
   (b) History of Numerical Methods
   (c) Overview of Manuscript
3. Herbivore Food Enhanced Predator-Prey Model.
   (a) Meaning of Coefficients
   (b) Determination of Values of Coefficients
4. GHC Model Results
   (a) Analysis of Equilibrium Populations (Fixed Points).
   (b) RK4 Numerical Solution to GHC Model.
5. GH Model
   (a) Interpretation of Model
   (b) Euler Numerical Solution with Spreadsheets
   (c) RK4 Numerical Solution to GHC Model.
6. G Model
   (a) Explicit Results
7. Discussion/Conclusion.
   (a) Overall conclusions
   (b) Improved model
8. Appendices.
   (a) RK4 Program
   (b) Euler Spreadsheet (Formulas)
   (c) Document Word Count
9. References.

Of course your report may have different section titles, but all important content must be included.

The **Title Page** should have a title, your name, the name of this course, the name and address of the school, and the date. The **Abstract** should be three to eight sentences overviewing the contents of the report. The abstract should be on a cover page which includes your name, your affiliation (i.e. your university), and the date. "Math 323 Report" is NOT an acceptable title for your manuscript, it should reflect the predator-prey-grass combination you chose.
The **Introduction** should have put the predator-prey problem in historical context (i.e. discuss Malthus, Lokta, Volterra, and others). Include a brief historical sketch of Runge, Kutta, Euler and when and why they introduced their methods. The introduction should also include an outline of the rest of the manuscript (in prose).

The **Theory** should include the derivation of the equations used in your model and be a discussion of the model used. Be sure to include the final equations used for your simulation.

The **Results** should contain graphs showing your solution and have a discussion of the numerical solution. You may need several subsections.

The **Discussion** Discuss the validity of your model (BOTH strengths and weaknesses). Also discuss other possible models. You will have to make several assumptions. These should be stated either here, or as the assumptions are used. You should also discuss other effects which you didn't but could include should contain a paragraph or two summarizing your results.

The **Appendix** should contain your C program(s) and comments on your numerical scheme. If there are other appendices, they should be included here as well. If there is only one appendix, it should be called "Appendix." Otherwise, they should be titled "Appendix A," "Appendix B," ...

You should use the numerical **Reference** system. See if you can find some others. Include at least 8 references, no more than 3 should be from wikipedia. Here's a good guide to the numerical reference system:
`https://www.dit.ie/media/library/documents/Numeric.pdf`
To create hyperlink in latex, use the \url{http://www.yahoo.com} command and include the hyperref package.