Eastern Oregon University
Homework #1
MATH 323 - Math Modeling

Name: _____
Major: _____
(staple this page to the front of your completed assignment as a cover page)

1. **Asymptotic and Piecewise Algebraic Modeling**
   Represent the function descriptions below in two ways:
   - In terms of the Heaviside step function $H(x)$
   - In terms of a continuous approximation,

$$h(x) = \frac{1}{2} + \frac{(1/2)x}{\sqrt{1/a^2 + x^2}} \tag{1}$$

(a)

$$f(x) = \begin{cases} 0 & x < 0 \\ x^2 & x \geq 0 \end{cases} \tag{2}$$

(b)

$$f(x) = \begin{cases} (x+3)^2 & x < -3 \\ 0 & -3 \leq x \leq 3 \\ (x-3)^2 & x > 3 \end{cases} \tag{3}$$

(c)

$$\lim_{x \to -\infty} f(x) = 3x, \lim_{x \to \infty} f(x) = 2x \tag{4}$$

(d)

$$\lim_{x \to -\infty} f(x) = 3x + 4, \lim_{x \to \infty} f(x) = 2x + 7 \tag{5}$$

(e)

$$\lim_{x \to -\infty} f(x) = mx + b, \lim_{x \to \infty} f(x) = m_2 x + b_2 \tag{6}$$

2. **Step and Pulse Function Models**
   (If you need help with an integral, ask maple)

   (a) If a pulse if given by

$$p(x) = \frac{1}{(1 + a^2 x^2)^{5/2}} \tag{7}$$

   what is the corresponding continuous step function, $h(x)$? Plot $p(x)$ when a is 1, 3, and 5, all on the same graph. Plot $h(x)$ when a is 1, 3, and 5, all on the same graph.

   (b) If a continuous step function is given by

$$h(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1}(ax) \tag{8}$$

   what is the corresponding pulse function $p(x)$? Plot $h(x)$ when a is 1, 3, and 5, all on the same graph. Plot $p(x)$ when a is 1, 3, and 5, all on the same graph.

3. **Comparative Pulse Models using the Binomial Expansion**
The Lorentzian pulse function

$$L(x) = \frac{1}{1 + a^2 x^2} \tag{9}$$

and the Gaussian pulse function

$$G(x) = e^{-a^2 x^2} = \frac{1}{e^{a^2 x^2}} \tag{10}$$

can be thought of as being on ends of a pulse "spectrum" (OK, only Dr. T sees it this way!). Using a Taylor Series expansion for the denominator....

   (a) Show that $G(x) \approx L(x)$ by using a first-order (linear) approximation of the Taylor Series expansion.

   (b) Determine the second-order approximation of $G(x)$ using a quadratic approximation of the Taylor Series expansion.

   (c) Determine the third-order approximation of $G(x)$ using a approximation approximation of the Taylor Series expansion.

4. **Rational Decision Making - Purchases**
You want to buy an LG HDTV. Using prices from walmart.com, make a graph of price vs. TV size. By hand, circle all of the data points, cross out the "strictly worse" ones. Then use a pencil and a straightedge to make a bilinear fit to the data. Which TV is the best "bang for your buck?" Draw a large arrow to it on the graph. What features "come along" with the TV that some of the cheaper TVs do not have?

5. **2D Path Optimization**
Dr. T moves to 2nd and Spring in La Grande. He still wants to walk to work. Which route should he take if...

   (a) It's sunny and warm

   (b) It's sunny, but there is a chilly wind from the east.

   (c) It's cold and there is snow/ice on the ground.

   For each answer, include a map showing the route, and a discussion as to why you picked that route.