(1) **Evaluation of line integral in Cartesian coordinates.** For the vector field $\mathbf{F} = y\mathbf{a}_x - z\mathbf{a}_y + x\mathbf{a}_z$, find $\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \cdot d\mathbf{l}$ for each of the following paths from $(0,0,0)$ to $(1,1,1)$: **(a)** $x = y = z$ and **(b)** $x = y = z^3$.

(2) **Evaluation of line integral around a closed path in Cartesian coordinates.** Given $\mathbf{F} = xy\mathbf{a}_x + yz\mathbf{a}_y + zx\mathbf{a}_z$, find $\oint_C \mathbf{F} \cdot d\mathbf{l}$, where C is the closed path comprising the straight lines from $(0,0,0)$ to $(1,1,1)$, from $(1,1,1)$ to $(1,1,0)$, and from $(1,1,0)$ to $(0,0,0)$.

(3) **Evaluation of line integral in Cartesian coordinates.** For the vector field $\mathbf{F} = \cos y\ \mathbf{a}_x - x \sin y\ \mathbf{a}_y$, find $\int_{(0,0,0)}^{(1,2\pi,1)} \mathbf{F} \cdot d\mathbf{l}$ in each of the following ways: **(a)** along the straight-line path between the two points; **(b)** along the curved path $x = z = \sin(y/4)$ between the two points; and **(c)** without choosing any particular path. Is the vector field conservative or nonconservative? Explain.

(4) **Evaluation of line integral around closed path in cylindrical coordinates.** Given $\mathbf{A} = 2r \sin\phi\ \mathbf{a}_r + r^2\mathbf{a}_\phi + z\mathbf{a}_z$ in cylindrical coordinates, find $\oint_C \mathbf{A} \cdot d\mathbf{l}$, where C is the closed path comprising the straight line from $(0,0,0)$ to $(1,0,0)$, the circular arc from $(1,0,0)$ to $(1,\pi/2,0)$ through $(1,\pi/4,0)$, the straight line from $(1,\pi/2,0)$ to $(1,\pi/2,1)$, and the straight line from $(1,\pi/2,1)$ to $(0,0,0)$.

(5) find the divergence and curl for each of the following:

(a) $zx\mathbf{a}_x + xy\mathbf{a}_y + yz\ \mathbf{a}_z$

(b) $\cos y\mathbf{a}_x - x \sin y\ \mathbf{a}_y$

(c) $(e^{-r^2}/r)\mathbf{a}_\phi$ in cylindrical coordinates

(d) $2r \cos\theta\ \mathbf{a}_r + r\mathbf{a}_\theta$ in spherical coordinates

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Example 2.2 Evaluation of a closed surface integral

Let us consider the magnetic field

$$\mathbf{B} = (x + 2)\mathbf{a}_x + (1 - 3y)\mathbf{a}_y + 2z\mathbf{a}_z$$

and evaluate $\oint_S \mathbf{B} \cdot d\mathbf{S}$, where S is the surface of the cubical box bounded by the planes

$$\begin{array}{ll} x = 0 & x = 1 \\ y = 0 & y = 1 \\ z = 0 & z = 1 \end{array}$$

as shown in Fig. 2.9.

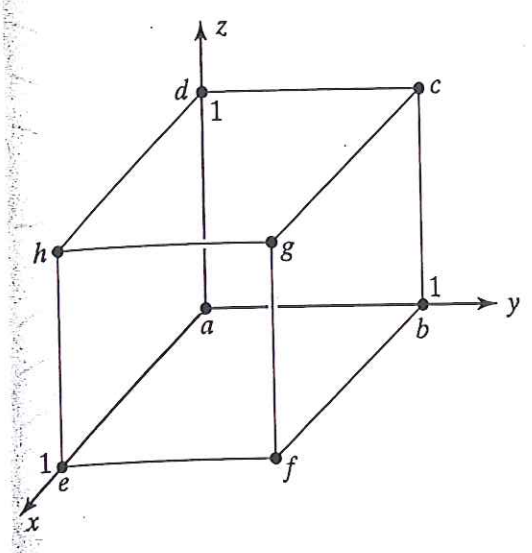

FIGURE 2.9

For evaluating the surface integral of a vector field over a closed surface.