Review

- Stewart 8.1-8.4, Sequences and Series
- Exam is Closed Book, 1 page of Notes
- Sequences
  - $\lim_{n\to\infty} a_n = L$, if L exists, sequence converges, else diverges
  - old limit laws still work
  - Squeeze Theorem
  - Terminology: Monotonic, Bounded

- Series
  - Geometric Series
    - $a(1+r+r^2+\dots+r^{N-1}) = \frac{a(1-r^N)}{1-r}$ (finite)
    - if $r<1$, $a\sum_{n=0}^{\infty} r^n = \frac{a}{1-r}$ (infinite)
  - $2.3\overline{17}$ as a rational number using series
  - Limit Theorem: $\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum a_n$ is divergent
  - Telescoping Series:

    $$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

  - Integral Test
    - If $\int_1^{\infty} f(x)\,dx$ is convergent, $\sum_{n=1}^{\infty} a_n$ is convergent
    - If $\int_1^{\infty} f(x)\,dx$ is divergent, $\sum$ = divergent
  - p-Series

    $$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ is } \begin{cases} \text{convergent} & p>1 \\ \text{divergent} & p \leq 1 \end{cases}$$

  - Note: If $p=1$, called a Harmonic Series

- Comparison Test ($\sum a_n, \sum b_n$ have positive terms)
  - If $\sum b_n$ is convergent and $a_n \le b_n \ \forall n$, then $\sum a_n$ is convergent
  - If $\sum b_n$ is divergent and $a_n \ge b_n \ \forall n$, then $\sum a_n$ is also divergent

- Limit Comparison Test
  - $\sum a_n$, $\sum b_n$ have only positive terms
  - If $\lim_{n\to\infty} \frac{a_n}{b_n} = C > 0$ and finite, then
    a_n converges $\Leftrightarrow$ b_n converges

- Alternating Series Test
  b_n's are positive
  If (1) $b_{n+1} < b_n \ \forall n$
  and (2) $\lim_{n\to\infty} b_n = 0$, then the series is convergent

- Alternate Series Estimation Theorem
  $S = \sum (-1)^{n-1} b_n$ satisfies
  ① $0 \le b_{n+1} \le b_n$
  ② $\lim_{n\to\infty} b_n = 0$, then $|R| = |S - S_n| \le b_{n+1}$

- Terminology: Absolute Convergence, Conditional Convergence

- Ratio Test
  $$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L = \begin{cases} <1 & a_n \text{ is abs. conv.} \\ =1 & \text{Test failed} \\ >1 & a_n \text{ is div.} \end{cases}$$

- Root Test
  $$\lim_{n\to\infty} \sqrt[n]{|a_n|} \text{ is } \begin{cases} <1 & a_n = \text{abs. conv.} \\ =1 & \text{Test failed} \\ >1 & a_n = \text{divergent} \end{cases}$$