Math 253 Cheat Sheet - Sequences and Series

Test	Notes
Geometric Series (finite)	$\sum\limits_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r} \quad (r \neq 1)$
Triangular Numbers	$\sum\limits_{i=0}^{n} i = \frac{n(n+1)}{2}$
Geometric Series (infinite)	$\sum\limits_{n=0}^{\infty} r^n = \begin{cases} \lvert r\rvert < 1 & \frac{1}{1-r} \\ \lvert r\rvert \geq 1 & Divergent \end{cases}$
Telescoping Series	$\sum\limits_{n=0}^{\infty} (a_n - a_{n+1}) = a_0 - a_\infty$
p-Series	$\sum\limits_{n=0}^{\infty} \frac{1}{n^p} = \begin{cases} p \leq 1 & Divergent \\ p > 1 & Convergent \end{cases}$
Test for Divergence	$\lim\limits_{x\to\infty} a_n = \begin{cases} 0 & No\,Info. \\ other & Divergent \end{cases}$
Alternating Series Test	$\sum\limits_{n=0}^{\infty} (-1)^n b_n$ where $b_n > 0 \;\forall\; n$ converges if $\begin{cases} (1) & b_{n+1} < b_n \forall n \\ (2) & \lim\limits_{n\to\infty} b_n = 0 \end{cases}$
Comparison Test (Compare $\sum\limits_{n=0}^{\infty} a_n$ to a known series $\sum\limits_{n=0}^{\infty} b_n$.)	$\begin{array}{c\|c\|c} & \text{Terms Get Smaller } (0 \leq a_n \leq b_n) & \text{Terms Get Bigger } (0 \leq b_n \leq a_n) \\ \hline \text{Known Conv. Series} & \text{Convergent} & \text{No Info.} \\ \text{Known Div. Series} & \text{No Info.} & \text{Divergent} \end{array}$
Limit Comparison Test (Compare $\sum\limits_{n=0}^{\infty} a_n$ to a known series $\sum\limits_{n=0}^{\infty} b_n$.)	$\begin{cases} If & a_n\ ,\ b_n \geq 0 \forall n? \\ and & \lim\limits_{n\to\infty} \frac{a_n}{b_n} = c > 0? \\ Then & \sum\limits_{n=0}^{\infty} a_n\ converges \iff \sum\limits_{n=0}^{\infty} b_n\ converges \end{cases}$
Ratio Test	$\begin{cases} \lim\limits_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L < 1 & Absolutely\ Convergent \\ \lim\limits_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L > 1 & Divergent \\ \lim\limits_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L = 1 & TestFails \end{cases}$
Root Test	$\begin{cases} \lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L < 1 & Absolutely\ Convergent \\ \lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L < 1 & Divergent \\ \lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L = 1 & TestFails \end{cases}$
Integral Test	$\begin{cases} If & a_n = f(n)\ is\ continuous,\ postive,\ and\ decreasing\ on\ (1,\infty) \\ Then & \int_1^\infty f(x)dx\ converges \iff \sum\limits_{n=0}^{\infty} a_n\ converges \end{cases}$

note: a little while doesn't matter: e^{-n^2}/n.

<u>Taylor Series</u>

$f(x) = \sum\limits_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ (McClaurin)

$f(x) = \sum\limits_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$

Pascal's Triangle:

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            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
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Applications: $(a+b)^n$, d^n $(a(x)\ b(x))$