Math 251 - Calculus I

HUMONGOUS Practice Final Exam
Eastern Oregon University

by

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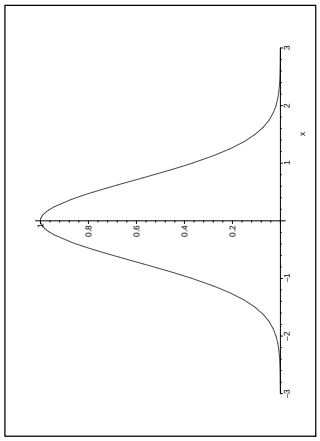
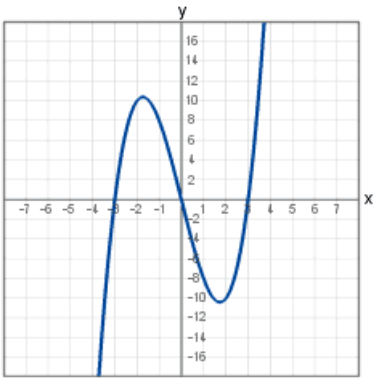
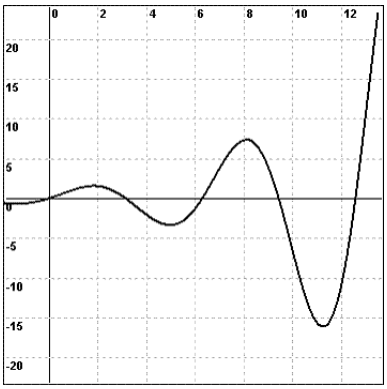
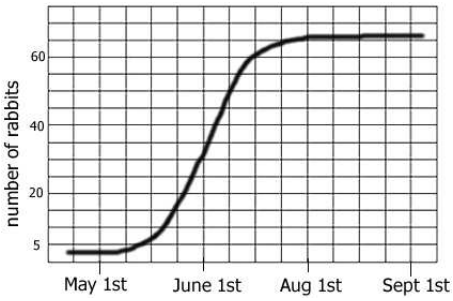
1. Classifying Functions from their Graphs
Consider the following graphs and any other you may wish to make up:









Answer the following about each graph:

(a) Is the function single-valued or multi-valued?

(b) Is this a continuous function?

(c) Is this a 1-1 function?

(d) Does this function have an inverse? If yes, graph it.

(e) Is the function monotonic increasing?

(f) Is the function monotonic decreasing?

(g) Is the function positive definite, positive semi-definite, negative definite, or negative semi-definite? If so, which?

(h) Is the function odd?

(i) Is the function even?

(j) Is the function periodic?

(k) Does the function have asymptotes? If so, what are the asymptotes?

(l) Does the function have extrema? If so identify them and specify if they are local mins or maxes.

(m) Identify regions where the graph is concave up.

(n) Identify regions where the graph is concave down.

(o) Identify the inflection points.

(p) Is this a single-variable function or a multi-variable function?

(q) What are the x-intercepts?

(r) What are the y-intercepts?

(s) Identify any fixed points

(t) Is the function bounded?

2. Classifying Functions and Graphing Them

Consider the following functions and any others you may wish to make up:

$$f(x) = x^6 + 4x^4 + 2x^2 + 8 \tag{1}$$

$$f(x) = (x+1)^2 sin(2x) \tag{2}$$

$$f(x) = (x-3)e^{2x} \tag{3}$$

$$f(x) = \frac{(x+1)^2}{x+2} \tag{4}$$

$$f(x) = 4^x tan(x) \tag{5}$$

For each of the functions answer the following:

(a) Find $f^{'}(x)$.

(b) Find $f^{''}(x)$.

(c) Find $f^{'''}(x)$.

(d) Does the function have extrema? If so, find the critical points. Are these critical points local maxima or minima?

(e) Find the regions where the function is increasing and where it is decreasing.

(f) Does the function have inflection points? If so, find them.

(g) Find the regions where the functions is concave up and where it is concave down.

(h) Are there any x-intercepts? If so, what are they?

(i) Are there any y-intercepts? If so, what are they?

(j) Are there any fixed points? If so and they are readily solvable, what are they?

(k) Is the function bounded?

(l) Does the function have asymptotes? If so, what are the asymptotes?

(m) What are the absolute minima?

(n) What are the absolute maxima?

(o) What is the domain of this function?

(p) Is this a 1-1 function? If not, give an example that proves it is not.

(q) Is the function odd or even (prove your answer)?

(r) Is the function periodic?

(s) Is the function single-valued or multi-valued?

(t) Is this a continuous function?

(u) Does this function have an inverse? If yes and it is readily solvable, determine the inverse function.

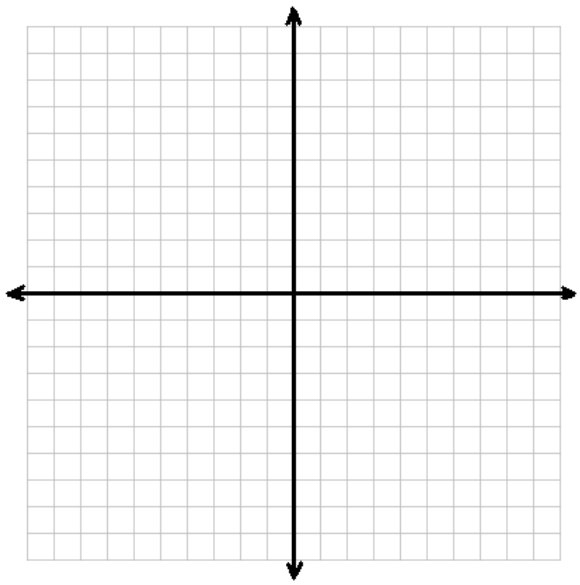
(v) Is the function monotonic increasing or monotonic decreasing?

(w) Is the function positive definite, positive semi-definite, negative definite, or negative semi-definite? If so, which?

(x) Using this information, graph the function.

(y) Is this a single-variable function or a multi-variable function?

(z) Graph the function:



3. Limit Definition of the Derivative
Use the limit definition of the derivative to find $f'(x)$ where $f(x)$ is given by the following functions (and any you wish to make up)

$$f(x) = \sqrt{x+3} \tag{6}$$

$$f(x) = x^3 + 3x + 4 \tag{7}$$

$$f(x) = \frac{x^2+2}{x+3} \tag{8}$$

4. Evaluating Limits Using L'Hospital's Rule
Use L'Hospital's Rule to evaluate each of the following limits:

(a) $\lim_{x\to 0} \frac{e^x-1}{x}$

(b) $\lim_{x\to 0} \frac{cos(x)-1}{x}$

(c) $\lim_{x\to 0} \frac{cos(x)-1}{x^2}$

(d) $\lim_{x\to 0} \frac{sin(x)}{x}$

5. First and Second Derivatives
Find $f'(x)$ and $f''(x)$ if $f(x)$ is given by the following:

(a) If $f(x) = x^4 + 3x^3 + 7x + 2$.

(b) If $f(x) = sin(x)$

(c) If $f(x) = sec(x)$

(d) If $f(x) = sech(x)$

(e) If $f(x) = sec^{-1}(x)$

(f) If $f(x) = sin(x^4 + 3x^3 + 7x + 2)$

(g) If $f(x) = sec(x^4 + 3x^3 + 7x + 2)$

(h) If $f(x) = sech(x^4 + 3x^3 + 7x + 2)$

(i) If $f(x) = sec^{-1}(x^4 + 3x^3 + 7x + 2)$

(j) If $f(x) = sin^4(x^4 + 3x^3 + 7x + 2)$

(k) If $f(x) = sin(sin(sin(x)))$

(l) If $f(x) = sec(sin(x))$

(m) If $f(x) = (x^4 + 3x^3 + 7x + 2)sin(x)$.

(n) If $f(x) = (x^4 + 3x^3 + 7x + 2)sec(x)$.

(o) If $f(x) = (x^4 + 3x^3 + 7x + 2)sech(x)$.

(p) If $f(x) = (x^4 + 3x^3 + 7x + 2)sec^{-1}(x)$.

(q) If $f(x) = sin(x)sech(x)$

6. Higher Order Derivatives (Pascal's Triangle)
Find $\frac{d^4}{dx^4} f(x)$ if

(a) $f(x) = (3x^2 + 7x + 2)cos(7x + 2)$.

(b) $f(x) = (4^x)cos(7x + 2)$.

7. Very High Order Derivatives and Antiderivatives
Find $\frac{d^{117}}{dx^{117}} f(x)$ and $\int^{(117)} f(x)dx^{117}$ if

(a) $f(x) = cos(7x + 2)$.

(b) $f(x) = sin(7x + 2)$.

(c) $f(x) = e^{7x+2}$.

(d) $f(x) = x^{100}$.

(e) $f(x) = x^{200}$.

8. Tangent and Normal Lines
Suppose $f(x) = x4^x - 2x^2 - 3$.

   (a) Determine the equation of the tangent line at (1,-1).

   (b) Determine the equation of the normal line at (1,-1).

9. Products of 3 or more functions (Logarithmic Differentiation)
Find $f'(x)$ by both successive application of the product rule and by Logarithmic differentiation (make sure your answers agree):

   (a) $f(x) = \sqrt{x+1}e^x cos(x)$

   (b) $f(x) = (x^3 + 8x + 9)\sqrt{x+1}4^x cos(3x^2 + 4)$

10. Anti-differentiation
Consider the following functions:

   (a) if $f(x) = cos(x)$, Find $\int f(x)dx$

   (b) if $f(x) = e^x$, Find $\int f(x)dx$

   (c) if $f(x) = x^3$, Find $\int f(x)dx$

   (d) if $f(x) = \frac{1}{x}$, Find $\int f(x)dx$

   (e) if $f(x) = \frac{4}{x} + sin(3x) + 6^x$, Find $\int \int f(x)dx^2$

11. Applications

(a) Implicit Differentiation

What is the slope of the tangent line of a unit circle at the point $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$?

(b) Optimization

Consider an object whose geometry consists of the wedge portion of a circle of radius r. If the internal angle of the wedge is ϕ then the arc length of the wedge is $r\phi$. The Area enclosed by the wedge would be

$$A = \pi r^2 \left(\frac{\phi}{2\pi}\right) \tag{9}$$

and the perimeter of the wedge is

$$P_0 = 2r + r\phi \tag{10}$$

If the perimeter is fixed, determine the angle ϕ that maximizes the area. Determine the corresponding radius, r, and area A. Draw the wedge. It's not a circle, why not?

(c) Classic Falling Ladder Problem

A ladder 15 ft long leans against a vertical wall. If the top slides down at 2 ft/sec, how fast is the angle of elevation of the ladder decreasing, when the lower end is 12 ft from the wall?
Solution
Video Explanation