Math 241 Cheat Sheet

by A. A. Tovar, Ph. D.

Linear Modeling

$$f(t) = f_0 + m(t - t_0)$$

Word Example

In the year 2000, there were 1200 EOU students:
$P(t) = 1200 + m(t - 2000)$

Data Example

First point: (t_1, P_1)
Last point: (t_2, P_2)
$P(t) = P_1 + m(t - t_1)$, where

$$m = \frac{P_2 - P_1}{t_2 - t_1}$$

Exponential Modeling

$$f(t) = f_0 e^{k(t-t_0)} = f_0 a^{(t-t_0)}$$

Note: $a = e^k$, $k = ln(a)$.
Note: $t_{doubling\ time} = (ln2)/k$, $t_{half\ life} = -(ln2)/k$.

Word Example

In the year 2000, there were 1200 EOU students:
$P(t) = 1200e^{k(t-2000)} = 1200a^{(t-2000)}$

Data Example

First point: (t_1, P_1)
Last point: (t_2, P_2)
$P(t) = P_1 e^{k(t-t_1)}$, where

$$k = \frac{1}{t_2 - t_1} ln\left(\frac{P_2}{P_1}\right)$$

Economics

Compounded Interest

$F = P(1 + i)^N$ (discrete compounding)
$F = Pe^{iN}$ (continuous compounding)

Cost and Revenue Functions ($C(q)$,$R(q)$)

$C(0) =$ Fixed Cost
$\frac{dC}{dq} =$ Marginal Cost $\approx C(q + 1) - C(q)$
$R(0) = 0$ (Fixed Revenue = 0)
$\frac{dR}{dq} =$ Marginal Revenue $\approx R(q + 1) - R(q)$
$\pi(q) = R(q) - C(q)$ (Profit)
Marginal Profit = Marg. Revenue - Marg. Cost
$p(q) = \frac{R(q)}{q}$ or $R(q) = pq$ (price/item × # items)
$a(q) = \frac{C(q)}{q} =$ Average Cost

Derivatives

Specific Functions

$\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)
$\frac{d}{dx}(e^x) = e^x$ (Exponential Rule)
$\frac{d}{dx}(ln(x)) = \frac{1}{x}$ (Logarithm Rule)

Examples:

$x^2 \overset{d/dx}{\Longrightarrow} 2x \overset{d/dx}{\Longrightarrow} 2 \overset{d/dx}{\Longrightarrow} 0 \overset{d/dx}{\Longrightarrow} 0$

$Cx^2/2 + c_1 x + c_2 \overset{d/dx}{\Leftarrow} Cx + c_1 \overset{d/dx}{\Leftarrow} C \overset{d/dx}{\Leftarrow} 0$

$\frac{1}{x} \overset{d/dx}{\Longrightarrow} \frac{-1}{x^2} \overset{d/dx}{\Longrightarrow} \frac{2}{x^3} \overset{d/dx}{\Longrightarrow} \frac{-6}{x^4} \overset{d/dx}{\Longrightarrow} \frac{24}{x^5} \left(\frac{(-1)^n n!}{x^{n+1}}\right)$

Notes:

$\sqrt{x} = x^{1/2}$, $\frac{1}{x^n} = x^{-n}$, $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{2/3}} = x^{-2/3}$

General Rules

$\frac{d}{dx}\left(af(x) + bg(x)\right) = a\frac{df}{dx} + b\frac{dg}{dx}$ (Superposition)
$\frac{d}{dx}\left(f(g(x))\right) = \frac{df}{dg}\frac{dg}{dx}$ (Chain Rule)
$\frac{d}{dx}\left(f(x)g(x)\right) = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$ (Product Rule)
$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{(g(x))^2}$ (Quotient Rule)

Graphing

Where	You Have...
$x = 0$	y-intercept
$f(x) = 0$	x-intercept (aka "zero" or "root")
$f'(x) < 0$	Function is Decreasing
$f'(x) = 0$	Critical Points
$f'(x) > 0$	Function is Increasing
$f''(x) < 0$	Function is Concave Down
$f''(x) = 0$	Point of Inflection
$f''(x) > 0$	Function is Concave Up

2nd Derivative Test: If x_{crit} is a critical point, then it is a maximum if $f''(x) < 0$. It is a minimum if $f''(x) > 0$.

Integrals

Power Rule for Integrals:

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Trapezoidal Rule (1 interval):

$$\int_a^b f(x)dx = \left(\frac{f(a) + f(b)}{2}\right)\Delta x$$

Trapezoidal Rule (many intervals):

$$\int_{x_0}^{x_n} f(x)dx = \left(\frac{f(x_0) + f(x_n)}{2} + f(x_1) + f(x_2) + f(x_3) + ... + f(x_{n-1})\right)\Delta x$$

Arithmetic Operations

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$(a+b)(c+d) = ab+ac+bc+bd$$

Exponents

$$a^b a^c = a^{b+c}$$

$$(a^b)^c = a^{bc}$$

$$a^n b^n = (ab)^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\sqrt{a} = a^{1/2}$$

$$\sqrt[n]{a} = a^{1/n}$$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} = a^{1/n}b^{1/n} = (ab)^{1/n}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{a^{1/n}}{b^{1/n}} = \left(\frac{a}{b}\right)^{1/n}$$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Factoring Formulas

$$x^2 - a^2 = (x+a)(x-a)$$

$$(x+a)(x+b) = x^2+(a+b)x+ab$$

$$(x+a)^2 = x^2 + 2ax + a^2$$

$$(x-a)^2 = x^2 - 2ax + a^2$$

$$x^3+a^3 = (x+a)(x^2-ax+a^2)$$

$$x^3-a^3 = (x-a)(x^2+ax+a^2)$$

Logarithms

$$log_b 1 = 0$$

$$log_b b = 1$$

$$log_b b^x = b^{log_b x} = x$$

$$log_b x^m = m log_b x$$

$$log_b(xy) = log_b x + log_b y$$

$$log_b\left(\frac{x}{y}\right) = log_b x - log_b y$$

Quadratic Formula

$$ax^2 + bx + c = 0$$

$$disc = b^2 - 4ac$$

$$x = \frac{-b \pm \sqrt{disc}}{2a}$$

Lines (Linear Functions)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

Parabolas (Quadratics)

$$y = ax^2 + bx + c$$
$$= a(x-h)^2 + k$$

$$Vertex\ @\ (h,k) = \left(-\frac{b}{2a}, -\frac{disc}{4a}\right)$$

$$a < 0 \Rightarrow \text{“opens down”}$$

$$a > 0 \Rightarrow \text{“opens up”}$$

Absolute Value Eqs

$$|f| = a \Rightarrow f = a \ \ or \ \ f = -a$$

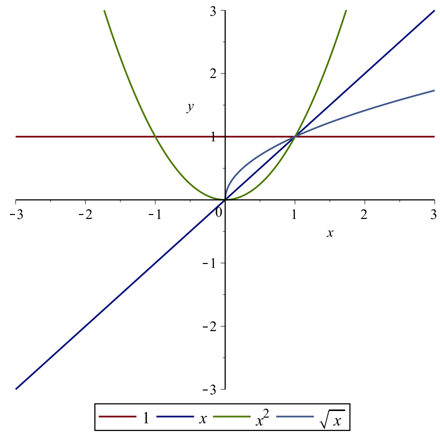
$$|f| < a \Rightarrow -a < f < a$$

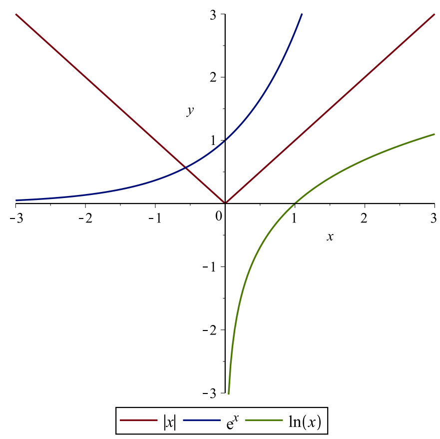
$$|f| > a \Rightarrow f < -a \ \ or \ \ f > a$$

Circle (Not a function!)

$$(x-h)^2 + (y-k)^2 = r^2$$

$$Vertex\ @\ (h,k),\ Radius = r$$





Tovar Notes:

1. $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
2. $(a+b)(c+d) = ab + ac + bc + bd$ is called the "foil method"
3. $a^b a^c = a^{b+c}$: bases are the same
4. $a^n b^n = (ab)^n$: exponents are the same
5. Next couple: what do negative exponents do? What do rational exponents do?
6. Distance Formula: 2 points, P_1 located at (x_1, y_1) and P_2 located at (x_2, y_2)