1. (4 points) Describe in your own words how Fourier analysis is useful in understanding musical sounds.

2. (6 points) Identify the function of the following:
   (A) The malleus, incus, and stapes combination.
   (B) The basilar membrane.
   (C) The hair cells.

3. (10 points) A tuning fork of frequency 384 Hz is held near the open end of a tube, which is open on both ends. The tube is adjustable in length, and the length is changed until a resonance is heard. This occurs at room temperature, where the speed of sound in air is 343 m/s.
   (A) What is the minimum possible length of the tube for a resonance to happen? Illustrate the resonance mode with a drawing showing nodes and antinodes.

   (B) Suppose we now make the tube longer. At what length will the next resonance happen? Draw another diagram illustrating the nodes and antinodes.
4. (8 points) Suppose that at an outdoor concert, one audio speaker is set up and produces sound with a SIL (sound intensity level) of 70 dB at a point 25 feet away from it.

(A) What would the SIL be if you placed another identical speaker beside the first, thus doubling the intensity? Would this make the perceived loudness twice as high?

(B) How many identical speakers would you have to set up side by side, so that the SIL is 80 dB (at the same place, 25 feet from the speakers.) In other words, you have raised the SIL by 10 dB.

5. (8 points) As we discussed in class, sound intensity often obeys a $1/r^2$ law: the intensity is proportional to $1/r^2$ where $r$ is the distance from the source.

Suppose a siren is on top of a pole and produces an intensity $I = 8.0 \times 10^{-4}$ W/m$^2$ at a point 100 feet away from the pole.

(A) What would the intensity $I$ be at a point 300 feet from the pole, assuming the $1/r^2$ law is obeyed?

(B) What is the SIL, in decibels, 100 feet from the siren?

(C) At the point 300 feet from the siren, has the SIL dropped by more than 10 dB or less than 10 dB, compared to the SIL 100 feet from the siren? Explain how you know.

6. (6 points) Suppose we play two “pure” sounds (sine waves), one at 250 Hz and the other at 254 Hz. The wave as a function of time is shown here. This illustrates the beats that occur:
Question: what will show up on a Fourier transform, such as we have done in class? Illustrate by completing the graph below.

7. (8 points) We show below a graph of constant loudness. Suppose you play a 1000 Hz tone at an intensity of 20 dB. Now play an 80 Hz tone. (A) What would the SIL level of the 80 Hz tone have to be, in order that it sound the same loudness as the 1000 Hz tone? (B) By what factor does the intensity, $I$, change? (In other words, what is the ratio of $I_{80}/I_{1000}$?) (C) Repeat for a 1000 Hz tone played at 80 dB instead of 20 dB.
SELECTED ANSWERS

1.

2. There are plenty of references for this.

3. (A)

   The tube length is $\lambda/2$, so we need to find $\lambda$.

   $\quad v = \lambda f$ so

   \[ \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{384 /\text{s}} = 0.89 \text{ m} \]

   \[ L = \frac{\lambda}{2} = 0.45 \text{ m} \]

   (B)

   The tube length is now $\lambda$, so $L = 0.89 \text{ m}$.

4. (A) When we double the intensity $I$, the SIL goes up by 3 dB. So the new SIL is $\beta = 73 \text{ dB}$.

   (B) To increase the SIL by 10 dB, you have to multiply the intensity $I$ by 10, so we need 10 speakers.

5. (A) The radius $r$ has increased by 3, so the new $I$ is $1/3^2$ times the old one.

   \[ I = \frac{8 \times 10^{-4} \text{ W/m}^2}{9} = 8.9 \times 10^{-5} \text{ W/m}^2 \]

   (B) The SIL is

   \[ \beta = 10 \log \frac{I}{I_0} = 10 \log \frac{8 \times 10^{-4} \text{ W/m}^2}{10^{-12} \text{ W/m}^2} \]

   \[ = 10 \log (8 \times 10^8) \]

   \[ = 89 \text{ dB} \]

   (C) It drops less than 10 dB. To drop by 10 dB, you have to divide the intensity $I$ by 10. But we only divided $I$ by a factor of 9.
6. 

Two peaks; one at 250 Hz and one at 254 Hz.

7. (A) Following the 20-phon line, we see it crosses 40 dB at 80 Hz.

(B) Each increase in 10 dB is a factor of 10 multiple. So

\[
\frac{I_{\text{at } 80 \text{ Hz}}}{I_{\text{at } 1000 \text{ Hz}}} = 100
\]

(C) It looks like the increase is from 80 dB to 87 dB, an increase of 7 dB. Since every 3 dB increase means doubling the intensity \( I \), \( I_{80}/I_{1000} \) is a bit more than 4; a **factor of 5** is a reasonable estimate.

Mathematically,

\[
7 \text{ dB} = 10 \log \frac{I_2}{I_1}
\]

\[
0.7 = \log \frac{I_2}{I_1}
\]

\[
10^{0.7} = \frac{I_2}{I_1}
\]

\[
\frac{I_2}{I_1} = 5.0
\]