Physics of Music
Homework 8
Solutions

Refer to the handout, “Notes on the decibel scale.” We will sometimes abbreviate sound intensity level as SIL.

1. What is the sound intensity level, in dB, of a sound with an “absolute” intensity of $2.5 \times 10^{-5}$ W/m$^2$?

Use the “plug-in” definition

$$\beta = 10 \log \frac{I}{I_{ref}}$$

$$= 10 \log \frac{2.5 \times 10^{-5}}{10^{12}} \text{ W/m}^2$$

$$= 10 \log (2.5 \times 10^7) \text{ W/m}^2$$

$$= 74 \text{ dB}$$

2. What is the intensity, in W/m$^2$, of a sound with a sound intensity level $\beta = 80$ dB?

This is an “even” power of 10 so we should not have to solve any equations. Remember that the threshold of pain is 120 dB and corresponds to $I = 1$ W/m$^2$. To get from there to 80 dB, we subtract 10 dB, 4 times. Each time we subtract 10 dB, we are dividing the intensity $I$ by 10. So we divide 1 W/m$^2$ by 10,000. This gives us $10^{-4}$ W/m$^2$.

3. Suppose a sound has a SIL of 40 dB. If I turn up the volume and increase it to 60 dB, by how much has the intensity (in W/m$^2$) been multiplied? What about a change from 30 dB to 50 dB? From 80 dB to 100 dB?

From 40 dB to 60 dB, we add 10 dB twice, so we are multiplying $I$ by 10 twice. We therefore multiplied the intensity $I$ by a factor of 100.

The same is true for a change from 30 dB to 50 dB, or 80 dB to 100 dB.

4. If I multiply the intensity (in W/m$^2$) by a factor of 10,000, by how much has the SIL in dB increased?

Multiplying intensity $I$ by 10, 4 times, adds 10 dB to the SIL, 4 times. Thus, the SIL increases by 40 dB.
5. If I multiply the intensity (in W/m\(^2\)) by a factor of 2, by how much has the SIL in dB increased?

See the handout on decibels. Increasing \(I\) by a factor of 2 adds \(3\,\text{dB}\) to the SIL.

6. What is the basilar membrane of the ear? To answer this, it is useful to visit a site like http://www.earaces.com/anatomy.htm. You can find lots of other resources using a “Google” search.

There are lots of resources for this.

7. The ear canal (from the outside of the ear to the eardrum) has a length of about 2.7 cm. Find the resonant frequency for the lowest frequency mode and for the mode of next highest frequency. Draw diagrams of nodes and antinodes to illustrate these modes.

In class we did this calculation and used it to explain the increase in sensitivity of the ear near 3500 Hz, as shown on the plot of equal loudness curves in your textbook (page 100). Summarize this argument.

For the lowest frequency, there is a node at the eardrum and an antinode at the opening, as shown in (A) below. Then the ear canal’s length is \(1/4\) of a wavelength. The wavelength for this resonance is then \(4(2.7\,\text{cm})\), or \(0.108\,\text{m}\). The frequency is then

\[
\nu = \frac{343\,\text{m/s}}{0.108\,\text{m}} = 3200\,\text{Hz}
\]

This is close to the “dip” in the curve noted. **NOTE: we do not give a number like “3176 Hz”**. The resonance is not very sharp, so it is silly to give the frequency to the nearest 1.0 Hz!

To find the next highest resonant frequency, we look at (B) in the diagram below, showing the node-antinode pattern for displacement. Clearly the wavelength is \(1/3\) of the previous value. If the wavelength is divided by 3, then the frequency must be multiplied by three to keep the velocity the same in the equation \(v = f\lambda\). Therefore the next resonant frequency is \(3(3200\,\text{Hz})\) or roughly 9600 Hz.