1. (5 points) Some electron microscopes accelerate their electrons through 80 KV of potential. That means that the electrons have a kinetic energy of \(8.0 \times 10^4\) eV. Find the de Broglie wavelength of these electrons. You do not need to find a relativistic momentum: for these energies, the classical expression will do. (Actually, the classically calculated velocity is too high, but the momentum calculated using \(p = mv\) is fairly close to the correct value.)

A hydrogen atom has a diameter of about 0.1 nm. How many wavelengths of these electrons would fit across the diameter of a H atom? Now, how does one of these wavelengths compare to the size of the nucleus of a H atom (that is, a proton)?

We use the de Broglie relation for wavelength:

\[
\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{mvc} = \frac{hc}{(mc^2)(v/c)}
\]

We need the velocity, which we find with the classical expression:

\[
KE = \frac{1}{2}mv^2
\]

\[
v = \sqrt{\frac{2(KE)}{m}}
\]

\[
\frac{v}{c} = \sqrt{\frac{2(KE)}{mc^2}} = \sqrt{\frac{2(8.0 \times 10^4 \text{ eV})}{5.11 \times 10^4 \text{ eV}}} = 0.5596
\]

Now we can find the wavelength:

\[
\lambda = \frac{hc}{(mc^2)(v/c)} = \frac{1240 \text{ eV-nm}}{(5.11 \times 10^5 \text{ eV})(0.5596)} = \frac{0.0043 \text{ nm}}{}\]

How many would fit across a H atom? \((0.10 \text{ nm})/(0.0043 \text{ nm}) \approx 23\) wavelengths.

However, the nucleus is a lot smaller: \(10^{-15}\) m or \(10^{-6}\) nm. So 4300 nuclei would fit in one of these wavelengths.

Let us compare by doing it the "right" way. We have

\[
\lambda = \frac{hc}{pc}
\]
and there is a relativistic relation

\[ E^2 = (pc)^2 + (mc^2)^2 \]

The total energy \( E \) of these electrons is the mass energy plus the kinetic energy:

\[ E = 511 \text{ KeV} + 80 \text{ KeV} = 591 \text{ KeV} \]

Therefore

\[ pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{(591 \text{ KeV})^2 - (511 \text{ KeV})^2} = 296.9 \text{ KeV} \]

Then

\[ \lambda = \frac{hc}{pc} = \frac{1240 \text{ eV-nm}}{2.969 \times 10^6 \text{ eV}} = 0.00418 \text{ nm} \]

Note than we could find the electron’s speed from

\[ p = \frac{mv}{\sqrt{1 - v^2/c^2}} \]

From this, you may obtain the result

\[ \frac{v}{c} = \frac{pc}{\sqrt{(pc)^2 + (mc^2)^2}} = 0.5024 \]

We see that our “classical” calculation for velocity is way off, but the wavelength result is not too bad.

2. (5 points) Repeat the previous problem for protons accelerated through an 80 KV potential. You might conclude that it would be better to use protons, rather than electrons, in such a microscope. Maybe, but the technical problems are overwhelming; electrons are a lot easier to produce. Besides, protons may be much more damaging to the samples being examined.

The mass energy of a proton is 938.3 MeV. So

\[ v = \sqrt{\frac{2(KE)}{m}} \]

\[ \frac{v}{c} = \sqrt{\frac{2(KE)}{mc^2}} = \sqrt{\frac{2(8.0 \times 10^4 \text{ eV})}{9.383 \times 10^8 \text{ eV}}} = 0.0131 \]

\[ \lambda = \frac{hc}{(mc^2)(v/c)} = \frac{1240 \text{ eV-nm}}{(9.383 \times 10^8 \text{ eV})(0.0131)} = 0.01 \times 10^{-4} \text{ nm} \]

Thus, about \( 1000 \) wavelengths would fit across the width of a H atom.
3. (4 points) A student calculates the energy of a 0.07078-nm photon as follows:

\[ E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J})(3.00 \times 10^8 \text{ m/s})}{(7.078 \times 10^{-11} \text{ m})(1.60 \times 10^{-19} \text{ J/eV})} = 17563 \text{ eV} \]

This is incorrect: the correct answer is 17520 eV, to four significant figures. Explain where the student went wrong in this calculation, and show how to do it correctly.

This student did not use enough significant figures for the constants he used in the calculation. If you are going to claim 5 significant figures in the answer, you must use at least 5 in all the numbers used for the calculation. (This should be elementary by now.) We have only 4 significant figures in the wavelength, so we can only keep that many in the result. The correct calculation would be

\[ E = \frac{hc}{\lambda} = \frac{(6.626076 \times 10^{-34} \text{ J})(2.99792458 \times 10^8 \text{ m/s})}{(7.078 \times 10^{-11} \text{ m})(1.6021773 \times 10^{-19} \text{ J/eV})} = 17517 \text{ eV, rounded to 17520 eV} \]

4. (4 points) What is the de Broglie wavelength of a honeybee (Apis melliera L, mass = 120 mg) flying at a speed of 2.0 m/s? Compare this to the diameter of a nucleus, about 1 femtometer.

The de Broglie relation is

\[ \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J}}{1.2 \times 10^{-4} \text{ kg}(2.0 \text{ m/s})} = 2.8 \times 10^{-30} \text{ m} \]

This is far smaller than a nucleus. In fact, this distance is about the same fraction of a nuclear diameter that a nuclear diameter is of 1 meter! That is, if you could, in thought, expand a nucleus to 1 meter in diameter, the honeybee’s wavelength is still about \(10^{-15}\) m, a nuclear diameter.
5. (4 points) In his book *Thirty Years that Shook Physics*, George Gamow explains why the theory of Rayleigh and Jeans failed to account for the observed blackbody radiation measurements. What analogy does Gamow use to explain this?

He used the analogy of a piano. Each string represents a “mode” of vibration of the atoms in the sides of the cavity. If equipartion held and there is no quantization, the energy ends up spread through all modes, with no limit on the energy density at the high frequencies.

6. (4 points) Before Bohr could devise his “orbital” theory for the hydrogen atom, our concept of the atom’s structure had to change. Who was responsible for this change, and what experiment was involved? (That is, how do we know that the “raison bread” or “plum pudding” model of Thomson is not correct?)

The Rutherford scattering experiment established the presence of a small nucleus of mass in the atom. This concept had to be there before Bohr could envision stable “orbits” around a central nucleus. The Thompson “plum pudding” model did not lend itself to this model — rather, it led to a harmonic oscillator type model, with only a single frequency for the hydrogen electron.