PHYS 202
Final Examination
March 16, 2004

<table>
<thead>
<tr>
<th>Problem</th>
<th>Page</th>
<th>Possible</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>Extra</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Clearly set up your solution with figures, lists of variables, what equation you’re using, etc., for maximum partial credit. Some useful equations and constants are given here.

Object | I
---|---
hoop | \(mR^2\)
cylinder | \(\frac{1}{2}mR^2\)
rod about end | \(\frac{1}{2}ml^2\)
solid sphere | \(\frac{1}{2}mR^2\)
hollow sphere | \(\frac{1}{3}mR^2\)

\[\rho = \bar{m} \bar{v} \quad \vec{F} \Delta t = \Delta \vec{p} \quad \text{Impulse} = \vec{F} \Delta t \quad W = W_s \quad W = \tau \theta\]
\[s = rt \quad v = r \omega \quad a = r \alpha \quad 180^\circ = \pi \text{ radians} \quad \Delta KE = W_{net}\]
\[KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad PE = mgh \quad PE = \frac{1}{2}kx^2 \quad F_g = \frac{GmM}{r^2} \quad a_c = v^2/r\]
\[\omega = \omega_0 + \alpha t \quad \omega^2 = \omega_0^2 + 2\alpha \theta \quad \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad L = I\omega \quad L = mvr \quad \text{(point mass)}\]
\[I = mR^2 \quad \tau = rF \sin \theta \quad \tau_{net} = I\alpha \quad F = -kx\]
\[x = A \sin \omega t \quad v = \omega A \cos \omega t \quad f = \frac{\omega}{2\pi} \quad T = 1/f = 2\pi /\omega \quad \omega = \sqrt{\frac{k}{m}} \quad \omega = \sqrt{\frac{g}{L}}\]
\[\omega = 2\pi f \quad E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad E_{total} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 \quad v = \lambda f \quad pV = NK T\]

Sound power in dB = \(10 \log \frac{I}{I_{ref}}\) where \(I_{ref} = 10^{-12} \text{ W/m}^2\). Sound: \(v_{air} = 343 \text{ m/s}\)

\[f_o = \left(\frac{1}{1-v_o/v}\right) f_s \quad \text{(moving source)} \quad f_o = (1 - v_o/v) f_s \quad \text{(moving observer)}\]
\[Q = mc \Delta T \quad Q = mL_f \quad L = L_o(1 + \alpha \Delta T) \quad V = V_o(1 + 3\alpha \Delta T) \quad \frac{\Delta V}{V_o} = \beta \Delta T\]
\[T_K = T_C + 273.16 \quad T_C = \frac{5}{9}(T_F - 32) \quad 1 \text{ cal} = 4.186 J \quad \beta = 3\alpha\]

<table>
<thead>
<tr>
<th>Substance</th>
<th>(c, J/(\text{kg}\cdot \text{C}))</th>
<th>(c, \text{ cal}/(\text{g}\cdot \text{C}))</th>
<th>(L_f, J/kg)</th>
<th>Coeff. of thermal exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>4190</td>
<td>1.00</td>
<td>3.33 \times 10^3</td>
<td>–</td>
</tr>
<tr>
<td>Oil (olive)</td>
<td>1970</td>
<td>0.47</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Iron</td>
<td>452</td>
<td>0.108</td>
<td>varies</td>
<td>(\alpha = 1.20 \times 10^{-5} \text{ C}^{-1})</td>
</tr>
<tr>
<td>Aluminum</td>
<td>920</td>
<td>0.22</td>
<td>3.80 \times 10^3</td>
<td>(\alpha = 2.5 \times 10^{-5} \text{ C}^{-1})</td>
</tr>
<tr>
<td>Copper</td>
<td>390</td>
<td>0.093</td>
<td>1.34 \times 10^3</td>
<td>(\alpha = 1.7 \times 10^{-5} \text{ C}^{-1})</td>
</tr>
<tr>
<td>Lead</td>
<td>130</td>
<td>0.031</td>
<td>2.45 \times 10^4</td>
<td>(\alpha = 2.9 \times 10^{-5} \text{ C}^{-1})</td>
</tr>
<tr>
<td>Ice</td>
<td>2100</td>
<td>0.50</td>
<td>–</td>
<td>(\alpha = 5.1 \times 10^{-5} \text{ C}^{-1})</td>
</tr>
</tbody>
</table>
1. (10 points) Suppose a hollow ball is given an initial velocity of 8.0 m/s at the bottom of an incline. The mass of the ball is 1.5 kg and its radius is 0.300 m. The incline makes an angle of 15° with the horizontal.

The ball rolls without slipping up the incline, and stops. What is the increase $h$ in the ball’s height by the time it stops? (The inertia of a hollow sphere is given on the front of this exam.)

Use conservation of energy, and include rotational kinetic energy.

Answer: 5.44 m
2. (12 points) Suppose we put a hole in a 2-meter-long rod of mass 4.0 kg, at a distance 0.80 m from one end. We hang the rod by a nail put through the hole, so that the rod may pivot freely about the nail.

(A) What is the moment of inertia $I$ of the rod, using the nail as the pivot point?

*Treat this as two rods, with $I = (1/3)mL^2$*

Answer: $I = 1.49$ kg-m$^2$.

(B) Suppose we lift the bottom end of the rod, so that it is horizontal. We then let go. What is the angular acceleration of the rod as it pivots about the nail, just after we let it go? Assume the center of mass is in the center of the rod.

*Use the rotational form of Newton’s second law: $\tau = I\alpha$.*

Answer: 5.25 rad/s$^2$

(C) When the rod passes through the vertical orientation, what is its angular velocity?

*Use conservation of energy.*

Answer: $\omega = 3.24$ rad/s.
3. (12 points) A violin “E” string has a length of 33.0 cm. Properly tuned, the frequency is 660 Hz for the fundamental.

(A) What is the speed of the waves along the string?

Answer: \( v = 436 \, \text{m/s} \)

(B) What would the frequency be if we touch the string 1/4 of the length from the end, forcing a nod there?

Answer: 2640 Hz.

(C) To produce the 660 Hz “E”, we must tighten the string so that its tension is 250 N. What is the mass of the 33-cm length of string?

Use the relation \( v = \sqrt{\frac{F}{\mu}} \)

\[ \mu = 1.32 \times 10^{-3} \, \text{kg/m}, \] giving a total mass of 0.434 grams.

(D) Suppose that when the string is vibrating in its fundamental mode, the “sideways” amplitude in the middle of the string is 2.0 mm: it moves back and forth that far. What is the maximum velocity of the middle point of the string (moving sideways)?

Use \( v_{\text{max}} = \omega A = 8.3 \, \text{m/s} \)
4. (6 points) A certain type of wooden flute acts like a tube open at both ends. If the flute is making a sound of frequency 489 Hz (a “B”), what is the length of the tube? Assume that this is the lowest possible frequency and that $v = 340 \text{ m/s}$ for sound.

*The length of the tube is one-half the wavelength when in fundamental mode.* Then using $v = 340 \text{ m/s}$, we find $\lambda = 34.8 \text{ cm}$.

5. (6 points) Suppose a stereo speaker is producing sound with an average intensity of $2.0 \times 10^{-6} \text{ W/m}^2$, measured at a distance 2.0 m from the speaker.

(A) What is this sound power level, expressed in dB?

Answer: 63 dB

(B) What would be the sound power level in dB a distance 8.0 m from the speaker, assuming that sound is radiated equally in all directions?

*Intensity is proportional to $1/r^2$. We increase $r$ by a factor of 4, so the intensity is divided by 16.*

$\beta = 51 \text{ dB}$. 
6. (12 points) A car tire of mass 8.0 kg is hanging from a rope, which is tied to a high limb of a tree. When a 25-kg child climbs onto the tire and sits on it, the tire is 20 cm lower than it is without the child.

(A) What is the spring constant for the system?

\[ k = \sqrt{\frac{\Delta F}{\Delta x}} = 1225 \text{ N/m} \]

or 1230 N/m

(B) Now, suppose the child jumps onto the tire, and begins oscillating up and down with an amplitude of 10 cm. What is the frequency of the oscillation, in Hz?

\[ \omega = \sqrt{\frac{k}{m}} = 6.09 \text{ rad/s} \]

\[ f = 0.97 \text{ Hz.} \]

(C) What is the total energy of the oscillating “system”?

\[ E = \frac{1}{2}kA^2 = 6.13 \text{ J} \]

(D) What is the maximum velocity of the tire during this motion?

Either use \( v_{\text{max}} = \omega A \) or use conservation of energy.

Either way, \( v_{\text{max}} = 0.61 \text{ m/s} \).
7. (10 points) American Indians used to boil water by dropping hot rocks into a container of water. Suppose 4.0 liters of water at 20.0° C is in an insulated container. A 0.80-kg rock at a temperature of 150° C is dropped into the water. Assuming no heat is lost and the system is allowed to equilibrate, find the final temperature of the system. The specific heat of a granite rock is 850 J/(kg·° C).

\[ \Delta Q_{water} + \Delta Q_{rock} = 0 \]
\[ m_{water}c_{water}(T_f - 20° C) + m_Rc_R(T_f - 150° C) = 0 \]

\[ T_f = 25.1° C \]

8. (5 points) Explain the difference between longitudinal waves and transverse waves. Give at least two examples of each.
9. (10 points) An aluminum cup of mass 300 grams contains 250 grams of water at 35° C. We then add 50 grams of ice at 0° C. What is the temperature of the system at equilibrium, after all the ice has melted?

\[
\Delta Q_{\text{ice}} + \Delta Q_{\text{water}} + \Delta Q_{\text{Al}} = 0 \\
m_{\text{ice}}L_f + m_{\text{ice}}c_{\text{water}}(T_f - 0^\circ C) + m_{\text{water}}c_{\text{water}}(T_f - 35^\circ C) + m_{\text{Al}}c_{\text{Al}}(T_f - 35^\circ C) = 0
\]

\[T_f = 19.3^\circ C\]

10. (4 points) We show below the paths of light rays traveling through rectangular blocks of glass immersed in various liquids. For which case is the index of refraction of the block less than that of the liquid? Explain how you know.

Optics problems will not be on the 2005 exam.
11. (8 points) We show here a beam of light incident on a face of a glass prism. The beam is moving parallel to the bottom of the prism (both are horizontal in the diagram.) The index of refraction for glass is 1.50.

(A) Draw a reasonable path for the beam as it enters and exits the prism.

(B) Determine the angle of refraction at the interface where the light enters the prism. Show on the diagram which angle this is.

![Diagram of light beam entering and exiting a glass prism at 45°](image)

12. (15 points extra credit) Do either one, but not both, on the back of this sheet.

(A) Refer to problem number 1, with the ball rolling up the incline. Determine the acceleration of the ball as it rolls up the ramp. Do this two ways: (1) Draw a free-body diagram of the ball. Obtain equations for $F_{net} = ma$ and $\tau_{net} = I\alpha$. Solve these simultaneous equations for $a$. (2) Just use $v_f^2 - v_i^2 = 2a(x - x_0)$. You get the same answer either way.

(B) Refer to problem number 2, with the 2-m rod. Suppose we let the rod oscillate near equilibrium. It is then a type of physical pendulum. Find the frequency of oscillation for small angles.