1. (6 points) Suppose we have cardboard tube which is 1.40 m long and 2 inches in diameter. Assume the speed of sound to be 343 m/s. We close the tube on one end, and leave the other end open.

(A) Find the lowest possible frequency for a standing sound wave in this tube. (This is called the fundamental frequency.)

As the diagram at the bottom shows, for the lowest frequency, the tube length is 1/4 of a wavelength. Thus, \( \lambda = 4\ell = 4(1.40 \text{ m}) = 5.6 \text{ m} \). Then since \( v = \lambda f \),

\[
f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{5.6 \text{ m}} = 61 \text{ Hz}
\]

(B) Determine the frequency of the first overtone.

The diagram at the bottom right shows the none-antinode pattern for the first overtone. From this, we see that \( \ell = (3/4)\lambda \). Then

\[
\lambda = (4/3)\ell = (4/3)(1.40 \text{ m}) = 1.87 \text{ m}
\]

We find that the frequency is

\[
f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{1.867 \text{ m}} = 184 \text{ Hz}
\]

Instead of doing this calculation, we could have simply noted that the wavelength for the first overtone is 1/3 of that for the fundamental. The frequency must therefore be three times higher. Then 3(61.3 Hz) = 184 Hz (keeping three significant figures everywhere.)

(C) For the fundamental, draw a diagram showing the node-antinode pattern of the standing wave. Repeat this for the first overtone.
2. (6 points) Suppose a guitar string is 0.80 m long and has a mass per unit length \( \mu = 3.00 \text{ grams/meter} \). This string (when strung on the guitar) has a fundamental frequency of 196 Hz (G below middle C).

(A) Determine the velocity of the waves on this string.

The velocity is related to wavelength and frequency by our fundamental relation \( v = \lambda f \). The wavelength is twice the string length, for the fundamental.

\[
v = \lambda f = (1.60 \text{ m})(196 \text{ Hz}) = 314 \text{ m/s}
\]

(B) Determine the tension in the string.

The velocity along the string is related to the tension and mass/length:

\[
v = \sqrt{\frac{F}{\mu}}
\]

\[
v^2 = \frac{F}{\mu}
\]

\[
F = v^2 \mu = (313.6 \text{ m/s})^2(0.0030 \text{ kg/m}) = 295 \text{ N}
\]

(C) What would the new frequency be if we increased the tension by 5%?

We note that \( v = \lambda f \) and \( v = \sqrt{F/\mu} \), so

\[
f = \frac{1}{\lambda} \sqrt{\frac{F}{\mu}}
\]

Since the wavelength and mass/length do not change, we may write

\[
f = (\text{constant}) \sqrt{F}
\]

Then to compare frequencies:

\[
f_1 = (\text{constant}) \sqrt{F_1}
\]

\[
f_2 = (\text{constant}) \sqrt{F_2} = (\text{constant}) \sqrt{1.05 F_1}
\]

Dividing the bottom equation by the top one, we get

\[
\frac{f_2}{f_1} = \sqrt{1.05}
\]

\[
f_2 = \sqrt{1.05} (196 \text{ Hz}) = 201 \text{ Hz}
\]
3. (3 points) A mandolin has 8 strings. It is tuned like a violin, with 4 notes: E-A-D-G, going down in fifths. The strings are tuned in pairs, so two of the strings are supposed to be tuned to an “A” at 440 Hz. Suppose you know that one of the strings actually is in tune, at 440 Hz. However, when you play the pair of strings together, you hear beats, at a frequency of 3 Hz. What are the two possible frequencies for the “out of tune” string?

We know that the beat frequency is the magnitude of the difference between the two frequencies: \( f_{\text{beat}} = |f_1 - f_2| \). Therefore the “unknown” frequency must be either 3 Hz above or 3 Hz below 440 Hz.

\[
 f = \begin{cases} 
 437 \text{ Hz} \quad \text{or} \\ 
 443 \text{ Hz}
\end{cases}
\]

4. (4 points) A microwave oven employs microwaves of frequency 2.45 GHz. These electromagnetic waves often set up standing waves inside the oven. If the standing waves are allowed to persist (and not broken up by some mechanism in the oven), the nodes would result in “cold spots” in whatever is being cooked.

What would be the distance between two nodes of such a standing wave? The speed of light is \( 3.00 \times 10^8 \text{ m/s} \).

The distance between any two nodes in a standing wave is half a wavelength. (This is a property of any standing wave.) We therefore need to know the wavelength of these microwaves.

\[
\lambda = \frac{v}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{2.45 \times 10^9 \text{ Hz}} = 0.122 \text{ m}
\]

The distance between nodes is half this, or \( 6.1 \text{ cm} \).

5. (5 points) You will need a spreadsheet to do this problem. The steps are these:

- Make a time series in column A: 0, 0.1, 0.2, etc, up to 10. Start this series in row 4.
- In cell B4, put in the formula \( =\sin(a4) \)
- Copy that formula down to cell B104 (the end of the time series.)

You now have a sine wave. The angular frequency of this wave is \( \omega = 1.0 \text{ rad/s} \). Do you see why? Excel and other spreadsheets usually read angles in radians, not in degrees. If you wish, make a plot of the wave for your own use.

Now, here is the problem: how would you generate a sine wave, in column C, which \textit{lags the wave in column B by 90^\circ} \? In other words, the second wave is “behind” the first one: when the first wave is passing through zero, the second wave is at -1. When the first wave is at 1, the second wave is passing through zero. When the first wave comes back down and is going through zero, the second wave is at 1. The two waves would look like this:
This diagram shows more complete cycles than your spreadsheet has. The diagram illustrates that if a complete cycle is $360^\circ$, the dotted-line wave is lagging the other one by $90^\circ$ (a quarter of a cycle.)

You do NOT have to turn in a plot. Just tell me what formula you should put in cell C4, to copy down in that column. Once you have the solution, you should verify (to yourself) that it works.

**SOLUTION.** In the expression $\sin \omega t$, a complete cycle occurs when the argument $\omega t$ changes by $2\pi$ radians or $360^\circ$. Now, $90^\circ$ corresponds to a quarter of a complete cycle, or $\pi/2$ radians. If the second wave is to be “behind” by this amount, we need to subtract $\pi/2$ radians from the argument of the original expression. In other words, if the original expression is

$$x = \sin (\omega t)$$

then the expression for the lagging wave is

$$x = \sin (\omega t - \pi/2)$$

In our present case, $\omega = 1.0 \text{ rad/s}$, so the spreadsheet expression for row 4 would just be

$$=\sin(A4 - \text{PI()/2})$$

Then you copy this down to the other rows. This works as predicted when we try it out.

You ought to recognize that if we added $\pi/2$ instead of subtracting it, the second wave would *lead* the first by that angle, instead of lagging. That is, the second wave would “peak” ahead of the first wave. You may verify this for yourself in the spreadsheet.