1. (5 points) Suppose a mass on a spring is vibrating up and down with an amplitude of 5.0 cm and a frequency of 0.80 Hz. (A) What is the angular speed $\omega$ of the system? 
(B) Describe the motion of the mass in the form

$$y = A \sin(\omega t)$$

(In other words, put in the values for $A$ and $\omega$.) (C) What is the maximum velocity of the mass, and what is the value of $y$ when this occurs? 

(A) We just use the relation between frequency and angular velocity:

$$\omega = 2\pi f = 2\pi (0.80 \text{ Hz}) = 5.0 \text{ rad/s}$$

(B) The amplitude is 5.0 cm, and we found $\omega$ above.

$$y = (5.0 \text{ cm}) \sin (5.0 \text{ rad/s} t)$$

2. (4 points) Show that a simple pendulum 1.0 meter long has a period of pretty close to 2.0 seconds. This is sometimes useful for measurements of time if one does not have a watch, since the bob’s motion from one side to the other takes very close to 1.0 second. 

Just use the expression for angular velocity of a simple pendulum:

$$\omega = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1.0 \text{ m}}} = 3.13 \text{ rad/s}$$

Now we can find the period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3.13 \text{ rad/s}} = 2.007 \text{ s}$$

or 2.0 seconds to two significant figures.
3. (4 points) Suppose a pendulum has a period of 4.00 s when swinging on the Earth. (Assume $g = 9.80 \text{ m/s}^2$.) What would be the period of the same pendulum if placed on the moon, where $g = 1.62 \text{ m/s}^2$? (Note: you do not really need to find the length of the pendulum. See if you can do this problem without that.)

We know that

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

which we can write

$$T\sqrt{g} = 2\pi \sqrt{\ell}$$

Now, the right side of the above equation does not change: it is a constant in this problem. Therefore $T\sqrt{g}$ is also constant, and we may write

$$T_1\sqrt{g_1} = T_2\sqrt{g_2}$$

Let “1” be the earth and “2” be the moon. Then

$$T_2 = T_1 \frac{\sqrt{g_1}}{\sqrt{g_2}} = (4.00 \text{ s}) \sqrt{\frac{9.80 \text{ m/s}^2}{1.62 \text{ m/s}^2}} = 9.84 \text{ s}$$

4. (5 points) Suppose a 70-kg person has been climbing a “climbing wall” in an athletic facility, and is hanging from a 15-m length of nylon rope. (She has let go of the supports on the wall.) The effective spring constant of the rope is $1.5 \times 10^4 \text{ N/m}$. The climber is oscillating up and down with an amplitude of 2.0 cm. (A) What is the frequency of oscillation? (B) What is the total energy of the climber/rope “system”? (We are only considering the energy of oscillation, without worrying about gravitational potential energy due to the climber’s height above the floor.)

(A) We use the expression for a mass on a spring:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{km} = \frac{1}{2\pi} \sqrt{\frac{1.5 \times 10^4 \text{ N/m}}{70 \text{ kg}}} = 2.3 \text{ Hz}$$

(B) At the ends of the oscillations, the total energy is kinetic in nature, since $v = 0$. Then

$$E = \frac{1}{2} kA^2 = \frac{1}{2} (1.5 \times 10^4 \text{ N/m})(0.020 \text{ m})^2 = 3.0 \text{ J}$$
Extra credit (3 points). It is not possible for the climber in this situation to oscillate with an amplitude of 5.0 cm, with simple harmonic motion. Show that this is a true statement. (Hint: There are at least two ways to do this. How much does the rope stretch when at equilibrium? What is the acceleration of the climber when she is at her maximum height in the cycle?)

Argument 1 When the climber first puts all his weight on the rope, how far does the rope stretch? We may determine this:

\[ F = kx \]
\[ x = \frac{F}{k} = \frac{mg}{k} = \frac{686 \text{ N}}{1.5 \times 10^4 \text{ N/m}} \]
\[ = 4.6 \text{ cm} \]

When the climber is not oscillating, he is sitting still at his equilibrium position, which is 4.6 cm below the “slack length” of the rope. If he rises more than 4.6 cm, the rope will go slack, and Hooke’s law would no longer work for the motion. So for a mass undergoing SHM, the amplitude could not be greater than 4.6 cm.

Argument 2 The net force on the climber is the vector sum of the upward force of the rope and the climber’s weight \( mg \). At the bottom of the cycle the rope can exert lots of force, so the upward acceleration there can be very great. However, at the highest point in the cycle, the maximum downward force can only be \( mg \), since the rope cannot push down. Therefore the climber’s maximum downward acceleration is just \( g \), free-fall. Now if the climber were undergoing SHM with an amplitude of 5.0 cm, what is his maximum acceleration? We find it using

\[ a_{\text{max}} = \omega^2 A = (2\pi f)^2 A \]
\[ = (2\pi(2.33 \text{ Hz}))^2(0.050 \text{ m}) \]
\[ = 10.7 \text{ m/s}^2 \]

Therefore, the maximum needed downward acceleration, when at the top of the cycle, is greater than free-fall acceleration. This can’t happen, so the climber cannot undergo SHM with this amplitude.
5. (5 points) Spider silk is extremely strong and can be stretched as much as 30% before breaking. (Compare this with 2% for a steel wire.) Suppose that when a 50 mg spider is hanging by his thread, the silk thread is 34.9 mm long. With no force applied, the same thread would be 30.0 mm long. (We have used published figures for spider silk strength, to determine this amount of stretch.)

(A) Find the force constant \( k \) for that thread, treating it as a spring. Keep two significant figures. (B) Suppose that something (like an air disturbance) causes the spider to begin oscillating up and down, like a mass on a spring. Find the frequency of the oscillation, in Hz.

(A) The force constant is

\[
k = \frac{\Delta F}{\Delta x} = \frac{mg}{\Delta x} = \frac{(5 \times 10^{-5} \text{ kg})(9.8 \text{ m/s}^2)}{0.0049 \text{ m}} = 0.10 \text{ N/m}
\]

(B) Since \( \omega = \sqrt{k/m} \), the frequency is

\[
f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{0.10 \text{ N/m}}{5.0 \times 10^{-5} \text{ kg}}} = 7.1 \text{ Hz}
\]

6. (2 points) What happens to the energy of a system vibrating with SHM, if you double the amplitude? (That is, by what factor does the total energy increase?)

The potential energy of a spring is \( \text{PE} = (1/2)kx^2 \). We know that the total energy of a system is \( (1/2)kA^2 \), since when \( x = A \), the kinetic energy is zero. So, the total energy of a system is proportional to \( A^2 \). If you double \( A \), you increase the total energy by a factor of 4.
7. (6 points) We show here a graph of a mass oscillating back and forth with some displacement. Use the graph to answer the following:

(A) What is the amplitude of the oscillation?
Looking at the graph, it is 2.0 cm.

(B) What is the angular velocity, \( \omega \)?
The period, \( T \), is 2.08 s, from the graph. Thus,

\[
\omega = \frac{2\pi}{T} = \frac{2\pi}{2.08 \text{ s}} = 3.0 \text{ rad/s}
\]

(C) Draw a line on the graph which is parallel to the curve as it passes through the equilibrium point. (Use a time when the curve’s slope is positive.) As you know, the slope of such a line will equal the instantaneous velocity at that time. In this case, it is the maximum velocity. Determine the slope of this line, finding a value for maximum velocity.

You can see our solution below. The heavy line goes through the curve near the equilibrium point. To make finding the slope easier, we drew a line parallel to this one, but beginning at time \( t = 0 \) (this is the dashed line.) We are going to use the full range of the graph, -2.5 cm to +2.5 cm, for finding the slope. To find the time value at the top, we drew a vertical (dotted) line down to the time axis. We see that the time to rise 5.0 cm is close to 0.85 seconds. Thus the slope is

\[
v = \frac{\Delta x}{\Delta t} = \frac{5.0 \text{ cm}}{0.85 \text{ s}} = 5.9 \text{ cm/s}
\]

(D) Use the expression \( v_{\text{max}} = \omega A \) to find the maximum velocity. Compare this value to the one you found in part (C)

\[
v_{\text{max}} = \omega A = (3.0 \text{ rad/s})(2.0 \text{ cm}) = 6.0 \text{ cm/s}
\]

This agrees closely with the value we found in (C). It’s probably more accurate, since in part (C) it is hard to judge precisely when the line is parallel to the curve.