1. (6 points) In the diagram below, each mass may be treated as a point mass equal to 1.00 kg. Find the rotation inertia of the system for axes as follows:

(A) rotational axis through masses B and C (as the left illustration),
(B) rotational axis through masses A and C (as the center illustration), and
(C) rotational axis in the center and perpendicular to the plane of the figure (as at right.)

(a) The rotational inertia of a point mass is $M r^2$ where $r$ is the distance from the axis of rotation. For this case, masses B and C are on the axis, so $r = 0$ for them; they do not contribute to the rotational inertia. The sum of the contributions from masses A and D is

$$I = \sum m_i r_i^2 = (1.0 \text{ kg})(1.0 \text{ m})^2 + (1.0 \text{ kg})(1.0 \text{ m})^2 = 2.0 \text{ kg-m}^2$$

(b) This time, masses A and C are on the axis, so they make no contribution to $I$. The distance from the axis to mass B or D is half the diagonal, which is $(1/2)\sqrt{2}(1.0 \text{ m}) = 0.707 \text{ m}$. Thus,

$$I = \sum m_i r_i^2 = (1.0 \text{ kg})(0.707 \text{ m})^2 + (1.0 \text{ kg})(0.707 \text{ m})^2 = 1.0 \text{ kg-m}^2$$

(c) All four masses contribute this time. The distance from the axis to each mass is the same as in part (b): 0.707 m. Therefore, each mass contributes the same to $I$ as each mass does in part (b). But here, there are four masses instead of two. So the rotational inertia will be twice what we calculated in part (b): $2.0 \text{ kg-m}^2$. (It is accidental that this is the same number as for part (a).)
2. (6 points) Suppose an adult bowling ball weighs 12 pounds and has some radius \( R \). This bowling ball has a moment of inertia \( I_o \) about its center.

(A) Suppose a child’s bowling ball has half the radius of the adult one, but is made of the same material. Find the rotational inertia of this ball, in terms of \( I_o \). (That is, do not express the inertia in absolute units like kg-m\(^2\).

(B) Suppose a toy bowling ball has half the radius of the adult one, and is made of a material whose density is 1/4 that of the “real” bowling ball. Find the rotational inertia of this ball, in terms of \( I_o \).

(A) Denote the rotational inertia of the adult’s ball as
\[
I_o = \frac{2}{5} M_o R_o^2
\]

First we need the mass, which is volume times density. The volume is proportional to radius cubed. (The equation is \( V = \frac{4}{3}\pi r^3 \), but all we really need is the proportionality.) The child’s ball has 1/2 the radius of the adult’s, so its volume is 1/8 of the adult’s. Since the density is the same, the mass must also decrease to 1/8 of the original: \( M_{\text{new}} = (1/8)M_o \). The child’s ball then has a rotational inertia
\[
I_{\text{new}} = \frac{2}{5} M_{\text{new}} R_{\text{new}}^2
\]
\[
= \frac{2}{5} \left(\frac{1}{8}\right) M_o \left(\frac{1}{2} R_o\right)^2
\]
\[
= \left(\frac{1}{32}\right) \frac{2}{5} M_o R_o^2
\]
\[
= \left(\frac{1}{32}\right) I_o
\]

Thus, the child’s ball has a rotational inertia 1/32\(^{nd}\) of the adult ball.

(B) If we reduce the density to 1/4 of that in the previous part, the mass goes down to 1/4 of the previous mass. Since rotational inertia is proportional to the mass, the new \( I \) value will be 1/4 of what we found in part (A). That is,

\[
I_{\text{new}} = \left(\frac{1}{128}\right) I_o
\]
3. (6 points) (A) Find the rotational inertia of a steel ball of radius 10 cm, about its center. Steel has a density of 7850 kg/m³. (B) Find the rotational inertia of a cube of steel with one side equal to 20 cm (the same as the diameter of the steel ball). The axis of rotation is perpendicular to one side and through the center of the cube.

(A) We know that the rotational inertia of a solid sphere is \( \frac{2}{5} MR^2 \). To find the mass, we multiply volume times density.

\[
M = V \rho = \frac{4}{3} \pi R^3 \rho \\
= \frac{4}{3} \pi (0.10 \text{ m})^3 (7850 \text{ kg/m}^3) \\
= 32.88 \text{ kg}
\]

Then the inertia is

\[
I = \frac{2}{5} MR^2 \\
= \frac{2}{5} (32.88 \text{ kg})(0.10 \text{ m})^2 \\
= 0.132 \text{ kg-m}^2
\]

(B) We know (from our handout on rotational motion or other source) that the rotational inertia of a square plate, with axis of rotation in the center and perpendicular to the plane, is \( \frac{1}{6} ML^2 \). We can consider the cube as a stack of plates. If so, the length of the side \( L \) does not change, and the mass is just the sum of the masses of all the plates. Therefore we can use this expression for the cube. The mass is just

\[
M = V \rho = L^3 \rho = (0.20 \text{ m})^3 (7850 \text{ kg/m}^3) = 62.8 \text{ kg}
\]

Then the rotational inertia is

\[
I = \frac{1}{6} ML^2 \\
= \frac{1}{6} (62.8 \text{ kg})(0.20 \text{ m})^2 \\
= 0.419 \text{ kg-m}^2 \quad (0.42 \text{ kg-m}^2 \text{ is also OK.})
\]
4. (6 points) A 3.6-m-long board of weight 200 N has a mass weighing 500 N sitting on it, as in the diagram. A fulcrum is 30 cm from the right end, and the mass's center of mass is 1.00 m from the fulcrum. A force $F$ is exerted upward, 30 cm from the left end of the board. The entire system is static: it is not moving. Assume that the board is uniform: its CM is at its center.

Find the value of the force $F$.

We use the fulcrum as a pivot point, and write the torque equation:

$$\tau_{CW} = \tau_{CCW}$$

$$(3.0 \text{ m})F = (1.5 \text{ m})(200 \text{ N}) + (1.0 \text{ m})(500 \text{ N})$$

$$F = \frac{800 \text{ N}}{3.0 \text{ m}}$$

$$F = \boxed{267 \text{ N}}$$

(270 N is also acceptable.)