

Practice Final Exam

Math 111

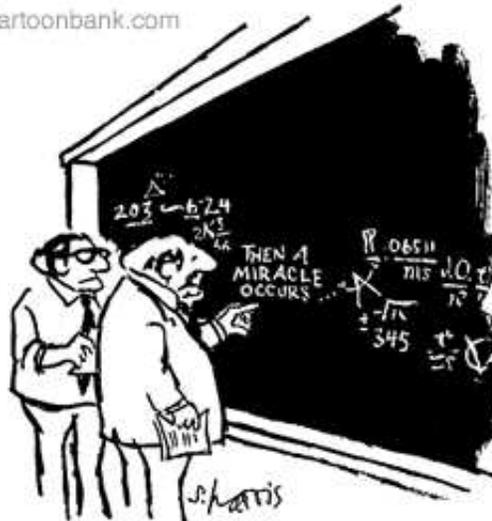
Instructor: Anthony Tovar, Ph. D.

Name: _____
Major: _____

Instructions:

1. The Examination will Closed Book, 1 page of Notes, Open Non-Graphing Calculator. You are to SHOW YOUR WORK, and BOX YOUR ANSWERS.
2. Make sure to Relax during the Final! That's when you're at your best!
3. This Practice Exam has 50 or so problems. I'm going to cut it down to 20 or so problems, change the equations a bit, and give it to you as your final. So, if you can do all of the problems here, you should do well on the final.

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"I think you should be more explicit here in step two."

1. Solve the following set of equations for x and y :

$$2x + 4y = 4 \quad (1)$$

$$3x - 2y = 2 \quad (2)$$

2. Solve the following set of equations for x and y :

$$5x + 2y = 9 \quad (3)$$

$$3x - 4y = 21 \quad (4)$$

3. Consider the number 135

- (a) Is it a whole number?
- (b) Is it an integer?
- (c) Is it a real number?
- (d) Is it an even number?
- (e) Is it an odd number?
- (f) Is it a prime number?
- (g) Is it a perfect square?
- (h) Is it a perfect cube?
- (i) Is it a power of 2?
- (j) Rewrite the number in terms of its prime factors?

4. Consider the number 64.

- (a) Is it a whole number?

- (b) Is it an integer?
- (c) Is it a real number?
- (d) Is it an even number?
- (e) Is it an odd number?
- (f) Is it a prime number?
- (g) Is it a perfect square?
- (h) Is it a perfect cube?
- (i) Is it a power of 2?
- (j) Rewrite the number in terms of its prime factors?

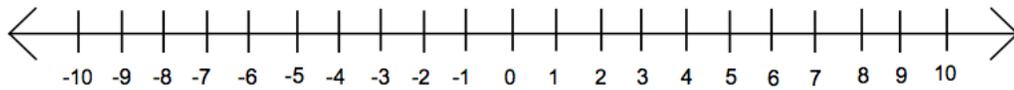
5. Solve the following equation for a :

$$\frac{3a - 1}{a^2 + 4a + 4} - \frac{3}{a^2 + 2a} = \frac{3}{a} \quad (5)$$

6. Suppose $A = [3, 8)$ and $B = (-7, 5]$.

(a) Below the following number line, graph A and B . Above the number line, graph $A \cap B$ and $A \cup B$ (Label your graphs).

(b) Write $A \cap B$ and $A \cup B$ as line intervals.



7. Solve the following inequality. Put your answer in both inequality notation and interval notation.

$$\left| \frac{1}{2}w - \frac{3}{4} \right| < 2 \quad (6)$$

8. For what value of x does each hold?

$$|3x + 5| = 2x + 6 \quad (7)$$

9. Write the following complex number in standard form:

$$\frac{i(3 - 2i)}{2 + 4i} \quad (8)$$

10. Write the following complex number in standard form:

$$i^{181} \quad (9)$$

11. Find the center and radius of the circle (box BOTH answers) given by the following equation (Hint: complete the squares for both x and y):

$$x^2 + y^2 - 2x - 6y = 39 \quad (10)$$

12. Find the equation of the line in the following scenarios. Put your answer in slope-intercept form.

(a) The slope of the line is 3. The y-intercept is -2.

(b) The slope of the line is -4. The line goes through (2,4).

(c) The line goes through (-3,2). The y-intercept is -1.

(d) The line goes through (1,-3). The line goes through (-2,2).

(e) The line is goes through (2,8) and is perpendicular to the line whose equation is $4x + 5y = 9$.

13. Find the domain of the function

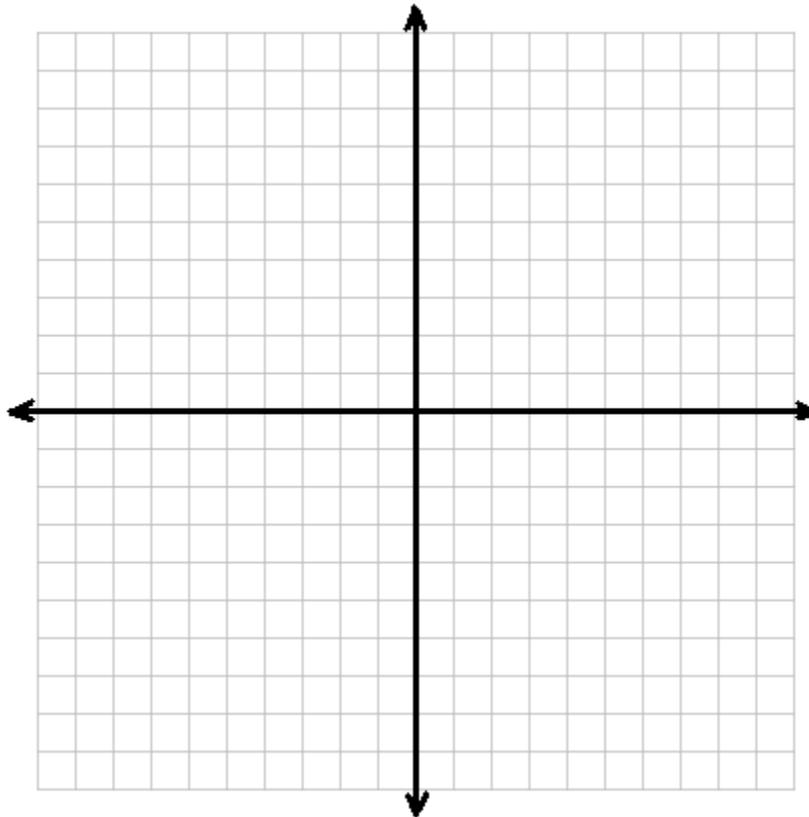
$$k(w) = \sqrt{7 + 2w} \quad (11)$$

(a) Specify your answer in interval notation.

(b) Specify your answer in inequality notation.

14. Graph the following function:

$$f(x) = \begin{cases} \frac{5}{2}x + 6 & : x < -2 \\ 1 & : -2 \leq x \leq 3 \\ \frac{3}{2}x - \frac{7}{2} & : x > 3 \end{cases} \quad (12)$$

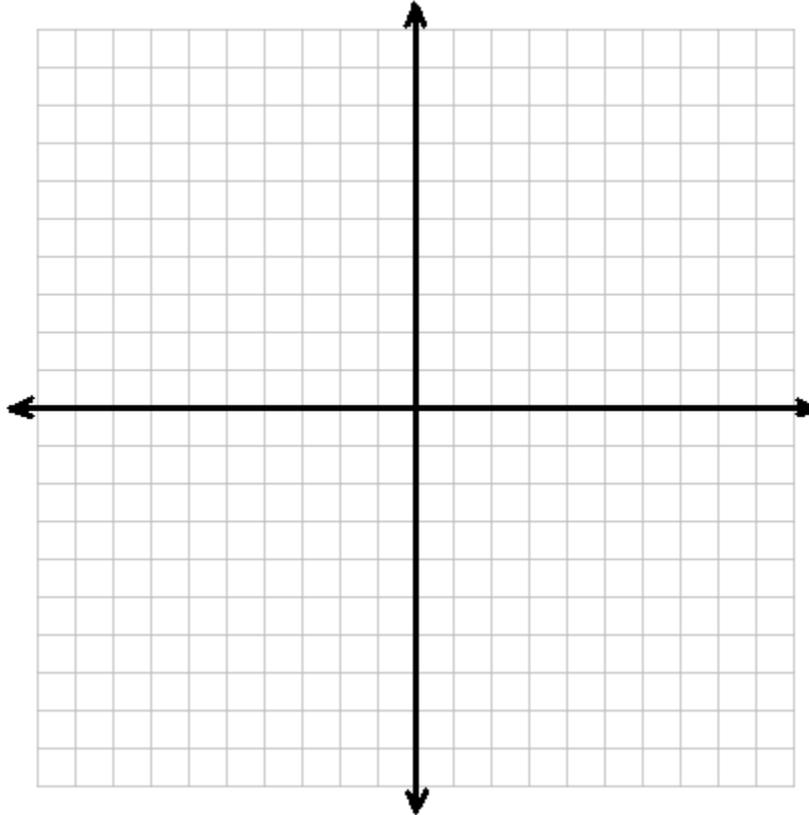


15. Inverse Functions

Consider the following function:

$$f(x) = \frac{1}{2}\sqrt{9-x} \tag{13}$$

(a) Graph the function, f



(b) Find the inverse function, f^{-1}

(c) On the previous graph, Add the line $y = x$ and the graph of f^{-1} .

(d) What is the Domain of f^{-1} ?

(e) What is the Range of f^{-1} ?

16. Inverse Functions

Find the inverse of

$$f(x) = \frac{2x^2}{x+1} \quad (14)$$

17. Polynomial Long Division

Use polynomial long division to reduce

(a)

$$(x^2 + 4x - 4) \div (x - 4) \quad (15)$$

(b)

$$\frac{2y^3 + 3y^2 + 3y - 1}{y + 2} \quad (16)$$

18. Variation and Modeling

- (a) Translate the following statement into an equation using k as the proportionality constant.

R varies jointly as the square of x and the square of y .

- (b) Translate the following statement into an equation using k as the proportionality constant.

The f-stop numbers N on a camera, known as the focal ratios, are directly proportional to the focal length F of the lens and inversely proportional to the diameter d of the effective lens opening.

- (c) If L is inversely proportional to the square of M . If $L = 9$ when $M = 9$, find L when $M = 6$.

19. Solve for x : $\log_3(x) = 2$

20. Simplify $\sqrt{2}^{\sqrt{2}} 2^{\sqrt{2}}$

21. Solve for x : $\log(x) - \log(2) = 2\log(x^3)$

22. Calculate $\log_{\sqrt{37}}(14.5)$ by rewriting the log with the weird base in terms of log base 10 and using your calculator.

23. Consider the following equation:

$$x^2 = 3x - 4 \quad (17)$$

(a) Solve the equation by factoring (Hint: put into standard form first. Don't just factor it, tell me the solutions.)

(b) Solve the equation by "Completing the Squares."

(c) Solve the equation by using the "Quadratic Formula."

24. **Transformation of Functions** (12 pts)

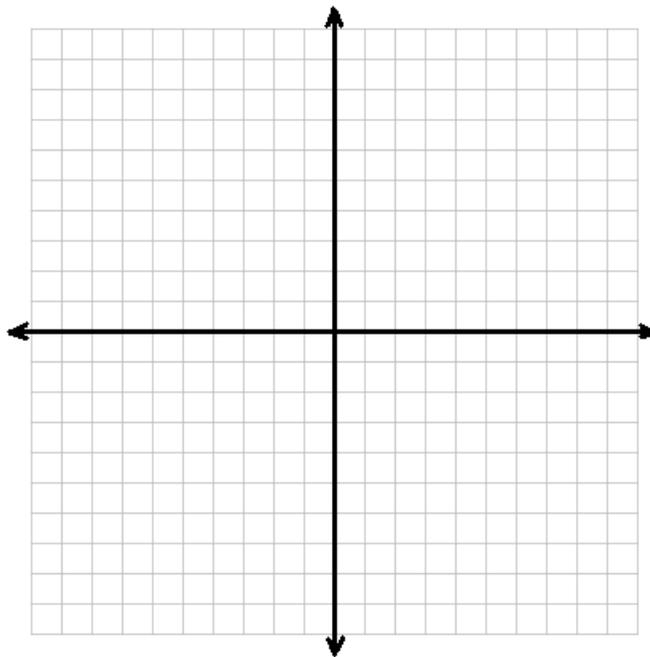
Graph the following

(a) $|x|$

(b) $|x + 3|$

(c) $-2|x + 3|$

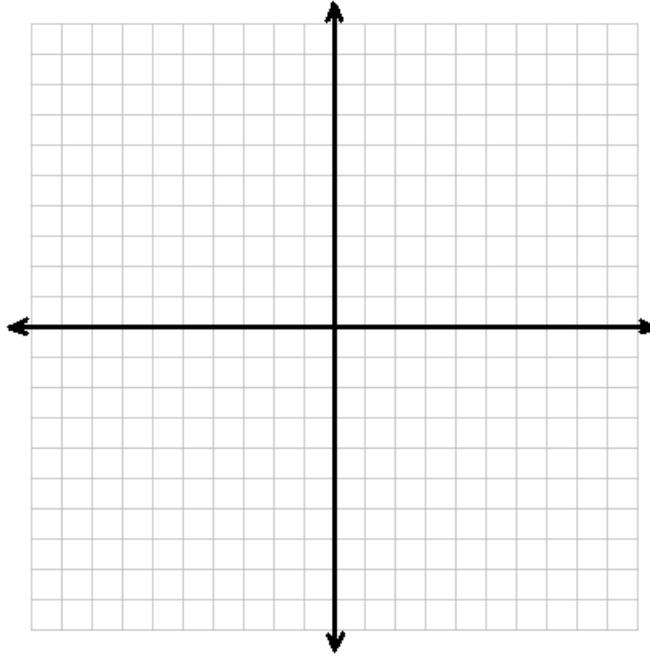
(d) $-2|x + 3| + 1$



25. **Transformation of Functions** (12 pts)

Graph the following

- (a) \sqrt{x}
- (b) $\sqrt{x-2}$
- (c) $-\sqrt{x-2}$
- (d) $-\sqrt{x-2} + 3$



26. **Quadratic Functions** (18 pts)

Consider the following function:

$$x^2 + 6x + 7 \tag{18}$$

- (a) Find the vertex point (Hint: complete the squares).

- (b) Determine the equation for the axis of symmetry.

- (c) What is the Domain of the function?

(d) What is the Range of the function?

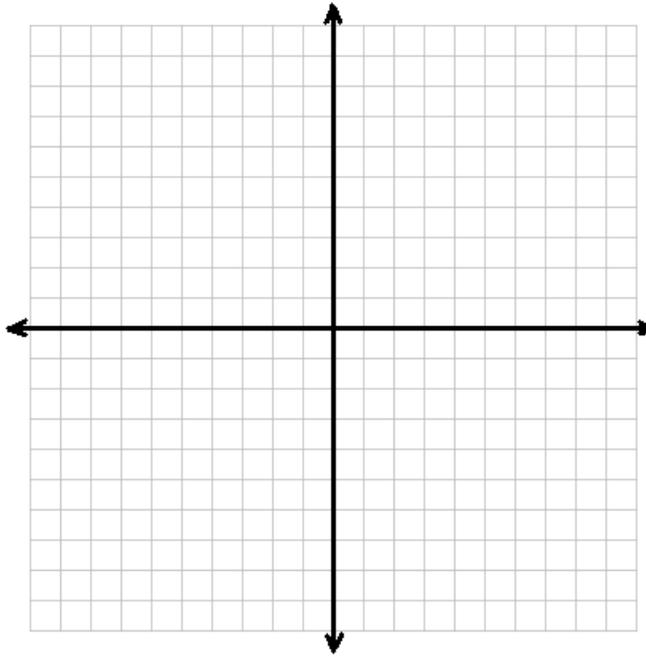
(e) Over what interval is the function increasing?

(f) Over what interval is the function decreasing?

(g) Determine the x-intercepts.

(h) Determine the y-intercept.

(i) Graph the Function on the axes below:



27. **Misc.** (18 pts)

- (a) The function $g(x)$ is $f(x)$ flipped upside down, then moved two units to the right, and 3 units down. Write g in terms of f .

- (b) Write the equation of a parabola opening up with center $(1,3)$.

- (c) Given an example (write an equation) of a parabola that opens up that has 2 roots.

- (d) Given an example (write an equation) of a parabola that opens down that has 2 roots.

- (e) Given an example (write an equation) of a parabola that opens up that has 1 root.

(f) Given an example (write an equation) of a parabola that opens up that has 0 roots.

(g) x^2 is an even function. x^3 is an odd function. What is \sqrt{x} ?

(h) Joe's weird function is a line of slope -1 that goes through the origin for all negative values of x and is a parabola that opens down and goes through the origin for all positive x . At x the value of the function is 2. Write the equation for Joe's weird function as a piecewise function.

(i) Consider the following polynomial: $3x^3 + x - 7$.

What is the degree of the polynomial? _____

What is the name of this type of polynomial? _____

You can't factor this polynomial, but you know that it can have at most _____ roots.

You know this last thing because of what theorem?

28. **Landscaping Application** (6 pts)

A rectangular garden plot is to be enclosed with a fence on three of its sides and a brick wall on the fourth side. If 100 feet of fencing material is available, what dimensions will yield the maximum area?

29. (5 pts) Find y such that $(2,y)$ is 3 units from $(-1,4)$.

30. (10 pts) Find the center and radius of the circle (box BOTH answers) given by the following equation (Hint: complete the squares for both x and y):

$$x^2 + y^2 - 6x - 4y = 36 \tag{19}$$

31. (15 pts) Find the equation of the line in the following scenarios. Put your answer in slope-intercept form.

(a) The slope of the line is 3. The y-intercept is -4.

(b) The slope of the line is -2. The line goes through (2,5).

(c) The line goes through (-1,-2). The y-intercept is -5.

(d) The line goes through (1,-2). The line goes through (-2,4).

(e) The line is goes through (3,8) and is perpendicular to the line whose equation is $3x + 4y = 9$.

32. (8 pts) Find the domain of the function

$$k(w) = \sqrt{7 + 3w} \quad (20)$$

(a) Specify your answer in interval notation.

(b) Specify your answer in inequality notation.

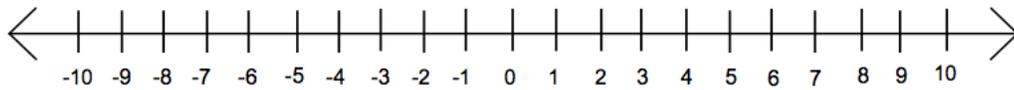
33. Reduce the following equation:

$$\left(\frac{x^2 - 16}{2x^2 + 10x + 8} \right) \div \left(\frac{x^2 - 13x + 36}{x^3 + 1} \right) \quad (21)$$

34. Suppose $A = [2, 9]$ and $B = (-8, 4)$.

(a) Below the following number line, graph A and B . Above the number line, graph $A \cap B$ and $A \cup B$ (Label your graphs).

(b) Write $A \cap B$ and $A \cup B$ as line intervals.



35. Solve the following inequality. Put your answer in both inequality notation and interval notation.

$$\left| \frac{1}{4}y - \frac{3}{2} \right| < 3 \quad (22)$$

36. For what value of x does each hold?

$$|3x + 5| = 2x + 6 \quad (23)$$

37. Write the following complex number in standard form:

$$\frac{1}{1 + 3i} \quad (24)$$

38. Write the following complex number in standard form:

$$(2x - 1) - (3y + 2)i = 5 - 4i \quad (25)$$

39. Write the following complex number in standard form:

$$i^{21} \quad (26)$$

40. **Logarithmic Expressions** (9 pts)

(a) Without using a calculator, evaluate $\log_{10} \sqrt[3]{10}$ (SHOW YOUR WORK!).

(b) Without using a calculator, evaluate $\log_3 \frac{1}{81}$ (SHOW YOUR WORK!).

(c) WITH a calculator, evaluate $\log_3 7$.

41. **Exponential Functions** (12 pts)

Reduce the following functions:

(a) $10^{3x-1}10^{4-x}$

(b) $(4^{3x})^{2y}$

42. **Exponential Function Equations** (8 pts)

Solve the following for x :

(a) $7^{x^2} = 7^{2x+3}$

(b) $9^{x^2} = 3^{3x-1}$

43. **Logarithmic Function Equations** (8 pts)

Solve the following for x :

(a) $\log x - \log 5 = \log 2 - \log(x - 3)$

(b) $1 - \log(x - 2) = \log(3x + 1)$

44. **Exponential Models** (4 pts)

It was estimated in 2008 that Kenya had a population of about 38,000,000 people, and a doubling time of 25 years. If growth continues at the same rate, find the population in

(a) 2012

(b) 2040

45. Solve the following equation:

$$3x = \sqrt{1 - 7x^2} \quad (27)$$

46. Solve the following equation:

$$2t^{-4} - 5t^{-2} + 2 = 0 \quad (28)$$

47. State whether the following functions are even, odd, or neither (prove your answer):

(a)
$$x^4 + 3x^2 \tag{29}$$

(b)
$$x^5 + 1 \tag{30}$$

48. Consider the following functions:

$$f(x) = x^2 + x, g(x) = \sqrt{x} \tag{31}$$

Determine

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $(f \circ f)(x)$

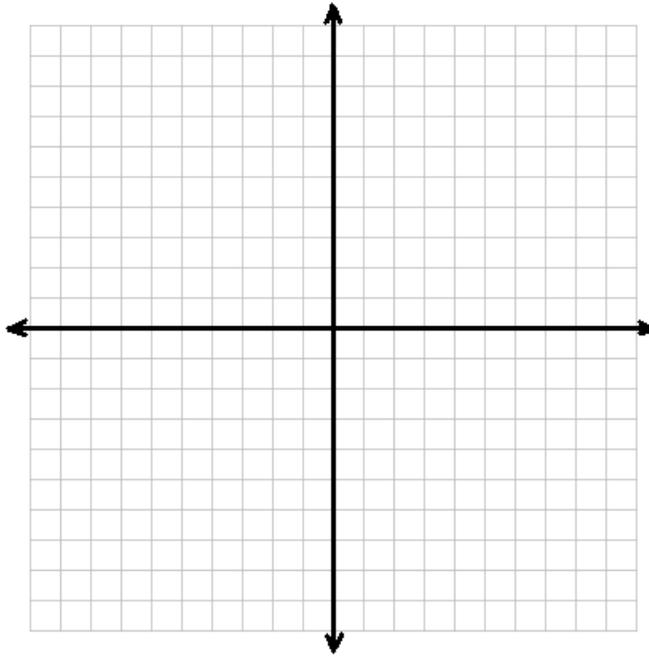
(d) $(g \circ g)(x)$

49. Consider the following function:

$$f(x) = \begin{cases} x^2 - 4 & : x \leq 2 \\ -2x + 4 & : x > 2 \end{cases} \tag{32}$$

(a) Find $f(-2)$, $f(0)$, $f(2)$, $f(4)$.

(b) Graph $f(x)$ on the axes below using the points you found above.



(c) Find the Domain.

(d) Find the Range.

50. Suppose

$$f(x) = x + \frac{1}{x} \quad (33)$$

$$g(x) = x - \frac{1}{x} \quad (34)$$

(a) Find $(f+g)(x)$ and state its domain

(b) Find $(f-g)(x)$ and state its domain

(c) Find $(fg)(x)$ and state its domain

(d) Find $(f/g)(x)$ and state its domain

51. (14 pts) **Polynomial Functions** Consider the following Polynomial:

$$P(x) = x^3 + 4x^2 - 11x - 30 \quad (35)$$

(a) What degree is this polynomial?

(b) What is the name of polynomials of this degree?

(c) Because the polynomial has this degree, it has at most how many solutions?

(d) You know this because of what theorem?

(e) By direct substitution, show that $P(3) = 0$, so that 3 is a root of $P(x)$.

(f) Determine $\frac{P(x)}{x-3}$.

(g) Determine all roots of $P(x)$

(h) Write $P(x)$ as a product three differences.

52. (6 pts) Ratio of Complex Numbers

Separate the following complex number into its real and imaginary parts, i.e. put the complex number in standard form:

$$\frac{5 + i12}{3 + i4} \quad (36)$$

53. (6 pts) Quadratic Equation Determine all of the solutions to

$$3x^2 + x + 7 = 0. \quad (37)$$

Be sure to reduce your answer.